Model Predictive Control Toolbox™ 3
Getting Started Guide

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Introduction

Product Overview

The Model Predictive Control Toolbox™ product is a collection of software that helps you design, analyze, and implement an advanced industrial automation algorithm. Like other MATLAB® tools, it provides a convenient graphical user interface (GUI) as well as a flexible command syntax that supports customization.

A Model Predictive Control Toolbox controller automates a target system (the *plant*) by combining a prediction strategy and a control strategy. An approximate linear plant model provides the prediction. The control strategy compares predicted plant signals to a set of objectives, then adjusts available actuators to achieve the objectives while respecting the plant constraints. Such constraints can include the physical limits of the actuator, boundaries of safe operation, and lower limits for product quality.

Model Predictive Control Toolbox constraint-tolerance differentiates it from other "optimal control" strategies (e.g., the Linear-Quadratic--Gaussian approach supported in Control System Toolbox™ software). The impetus for this is industrial experience suggesting that the drive for profitability often pushes the plant to one or more constraints. The Model Predictive Control Toolbox controller considers such factors explicitly, allowing it to allocate the available plant resources intelligently as the system evolves over time.

Model Predictive Control Toolbox software uses the same powerful linear dynamic modeling tools found in Control System Toolbox software and System Identification Toolbox™ software. You can use transfer functions, state-space matrices, or a combination of transfer functions and state-space matrices. You can also include delays, which are a common feature of industrial plants.

If you do not have a model but can perform experiments, System Identification Toolbox software can help you to develop a plant model from the data and then design a controller for this plant using the Model Predictive Control Toolbox product.

If you use Simulink® to model your plant, the Model Predictive Control Toolbox product provides a Simulink controller block. For example, you can linearize a nonlinear Simulink model, use the linearized model to build a Model Predictive Control Toolbox controller, and evaluate its ability to control the nonlinear model. If you determine that this controller performs well, you can implement this control strategy in a real plant using Real-Time Workshop® software.

For a list of books on predictive control theory and practice, see “Bibliography” on page 1-6. In particular, Maciejowski [4] illustrates and extends Version 1.0 of the Model Predictive Control Toolbox software. (The command format used in [4] is obsolete in Model Predictive Control Toolbox Version 2.0, however.)

Using the Documentation

If you have limited experience with MATLAB® or Model Predictive Control Toolbox™ software, read this guide first. It shows how to:

- Define your plant using Control System Toolbox™ modeling tools (LTI transfer function and state space models).
- Derive a linear plant model from a nonlinear Simulink® representation.
- Design Model Predictive Control for your plant using `mpctool`, the graphical user interface (GUI), or special commands.
- Simulate Model Predictive Control performance using `mpctool`, Simulink, or commands.

If you have experience with an earlier Model Predictive Control Toolbox release, we advise you to read this document to familiarize yourself with the many new features and the new command syntax (the earlier syntax is still available, but the underlying code is no longer supported).

If you need more details, see the online documentation. To access it from the MATLAB prompt, type:

```
helpdesk
```

When the help dialog box appears, select **Model Predictive Control Toolbox** in the **Contents** pane. This displays a roadmap with links to the available documentation components. Briefly, these are:

- Getting Started. The online version of this document.
- MPC Problem Setup. Mathematical details of the Model Predictive Control Toolbox algorithm and user specifications required for controller design.
- MPC Simulink Library. Describes the Model Predictive Controller Block and its use within Simulink.
- Case-Study Examples. Example toolbox applications.
- The Design Tool. Reference manual for `mpctool`, the GUI.
- Functions. Reference manual describing each Model Predictive Control Toolbox function (used for controller design and simulation in MATLAB commands and scripts).
- Blocks. Reference manual describing the Model Predictive Control Toolbox blocks (used for controller design and simulation in Simulink).

- Object Reference. Details of the controller object that represents a complete controller design.
- Release Notes. Summarizes major features of this release, known limitations, etc.

Related Products

The MathWorks™ provides other products that complement and enhance the Model Predictive Control Toolbox functionality. For more information, see `www.mathworks.com`.

Bibliography

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[5] Prett, D., and C. Garcia, *Fundamental Process Control*, Butterworths, 1988.

[6] Rossiter, J. A., *Model-Based Predictive Control: A Practical Approach*, CRC Press, 2003.

Building Models

Overview

This section covers the following topics:

- “Plant Model” on page 2-2
- “Plant Inputs and Outputs” on page 2-3

Plant Model

The *plant* is the system (process or device) you intend to control. Figure 2-1 shows a schematic example.

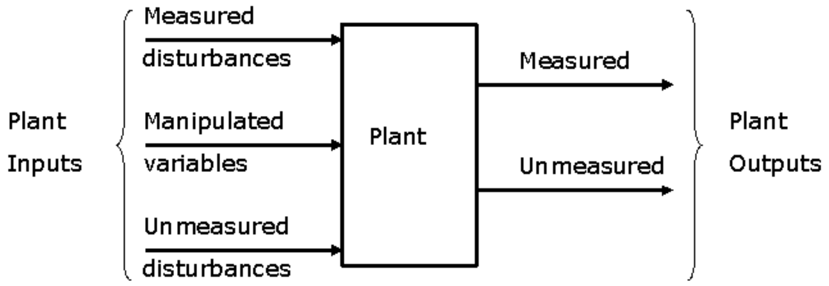


Figure 2-1: Plant with Input and Output Signals

A Model Predictive Control Toolbox™ design requires a *plant model*, which defines the mathematical relationship between the plant inputs and outputs. The controller uses it to predict plant behavior.

The toolbox software requires the model to be linear, time invariant (LTI). You can define such a model as follows:

- Create a transfer function, state space, or zero/pole/gain model using methods provided by the Control System Toolbox™ software
- Derive it from plant data using, e.g., methods provided by System Identification Toolbox™ software
- Derive it by linearizing a Simulink® model

This chapter illustrates each of these approaches. Control System Toolbox, Simulink, and System Identification Toolbox documentation provides additional examples and details.

Plant Inputs and Outputs

Inputs

The *plant inputs* are the independent variables affecting the plant. As shown in Figure 2-1, there are three types:

Measured disturbances. The controller can't adjust them, but uses them for feedforward compensation.

Manipulated variables. The controller adjusts these in order to achieve its goals.

Unmeasured disturbances. These are independent inputs of which the controller has no direct knowledge, and for which it must compensate

Outputs

The *plant outputs* are the dependent variables (outcomes) you wish to control or monitor. As shown in Figure 2-1, there are two types:

Measured outputs. The controller uses these to estimate unmeasured quantities and as feedback on the success of its adjustments.

Unmeasured outputs. The controller estimates these based on available measurements and the plant model. The controller can also hold unmeasured outputs at setpoints or within constraint boundaries.

You must specify the input and output types when designing the controller. See "Input and Output Types" on page 2-9 for more details.

Linear, Time Invariant (LTI) Models

Model Predictive Control Toolbox™ software supports the same LTI model formats as does Control System Toolbox™ software. You can use whichever is most convenient for your application. It's also easy to convert from one format to another.

The following sections describe the three model formats and the commands used to construct them:

- "Transfer Function Format" on page 2-4
- "Zero/Pole/Gain Format" on page 2-5
- "State-Space Format" on page 2-5
- "LTI Object Properties" on page 2-7
- "Multiinput Multioutput (MIMO) Plants" on page 2-11
- "LTI Model Characteristics" on page 2-13

For more details, see the Control System Toolbox documentation.

Transfer Function Format

A transfer function (TF) relates a particular input/output pair. For example, if $u(t)$ is a plant input and $y(t)$ is an output, the transfer function relating them might be:

$$\frac{Y(s)}{U(s)} = G(s) = \frac{s+2}{s^2+s+10}e^{-1.5s}$$

This TF consists of a *numerator* polynomial, $s + 2$, a *denominator* polynomial, $s^2 + s + 10$, and a delay, which is 1.5 time units here. You can define G using Control System Toolbox `tf` function:

```
Gtf1 = tf([1 2], [1 1 10], 'OutputDelay', 1.5)
```

Control System Toolbox software builds and displays it as follows:

```
Transfer function:
                    s + 2
exp(-1.5*s) * ------------
              s^2 + s + 10
```

Zero/Pole/Gain Format

Like the TF, the zero/pole/gain (ZPK) format relates an input/output pair. The difference is that the ZPK numerator and denominator polynomials are factored, as in

$$G(s) = 2.5\frac{s + 0.45}{(s + 0.3)(s + 0.1 + 0.7i)(s + 0.1 - 0.7i)}$$

(zeros and/or poles are complex numbers in general).

You define the ZPK model by specifying the zero(s), pole(s), and gain as in

```
Gzpk1 = zpk( -0.45, [-0.3, -0.1+0.7*i, -0.1-0.7*i], 2.5)
```

State-Space Format

Chemical Reactor Example

The state-space format is convenient if your model is a set of LTI differential and algebraic equations. For example, consider the following linearized model of a continuous stirred-tank reactor (CSTR) involving an exothermic (heat-generating) reaction [9]

$$\frac{dC'_A}{dt} = a_{11}C'_A + a_{12}T' + b_{11}T'_c + b_{12}C'_{Ai}$$

$$\frac{dT'}{dt} = a_{21}C'_A + a_{22}T' + b_{21}T'_c + b_{22}C'_{Ai}$$

where C_A is the concentration of a key reactant, T is the temperature in the reactor, T_c is the coolant temperature, C_{Ai} is the reactant concentration in the reactor feed, and a_{ij} and b_{ij} are constants. See the process schematic in Figure 2-2. The primes (e.g., C'_A) denote a deviation from the nominal steady-state condition at which the model has been linearized.

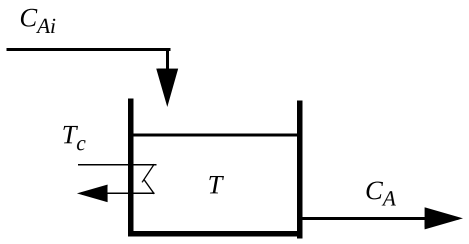


Figure 2-2: CSTR Schematic

Measurement of reactant concentrations is often difficult, if not impossible. Let us assume that T is a measured output, C_A is an unmeasured output, T_c is a manipulated variable, and C_{Ai} is an unmeasured disturbance.

State-Space Format

The model fits the general state-space format

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

where

$$x = \begin{bmatrix} C'_A \\ T' \end{bmatrix}, u = \begin{bmatrix} T'_c \\ C'_{Ai} \end{bmatrix}, y = \begin{bmatrix} T' \\ C'_A \end{bmatrix},$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The following code shows how to define such a model for some specific values of the a_{ij} and b_{ij} constants:

```
A = [-0.0285  -0.0014
     -0.0371  -0.1476];
B = [-0.0850   0.0238
      0.0802   0.4462];
```

```
C = [0 1
     1 0];
D = zeros(2,2);
CSTR = ss(A,B,C,D);
```

This defines a *continuous-time* state-space model. If you do not specify a sampling period, a default sampling value of zero applies. You can also specify discrete-time state-space models. You can specify delays in both continuous-time and discrete-time models. For more information, see the Control System Toolbox documentation.

Note In the CSTR example, the D matrix is zero and the output does not instantly responds to change in the input. The Model Predictive Control Toolbox software prohibits direct (instantaneous) feedthrough from a manipulated variable to an output. For example, the CSTR model could include direct feedthrough from the unmeasured disturbance, C_{Ai}, to either C_A or T but direct feedthrough from T_c to either output would violate this restriction. If the model had direct feedthrough from T_c, you could have added a small delay at this input to circumvent the problem.

LTI Object Properties

The ss function in the last line of the above code creates a state space model, CSTR, which is an *LTI object*. The tf and zpk commands described in “Transfer Function Format” on page 2-4 and “Zero/Pole/Gain Format” on page 2-5 also create LTI objects. Such objects contain the model parameters as well as optional properties.

LTI Properties for the CSTR Example

The following code sets some of the CSTR model’s optional properties:

```
CSTR.InputName = {'T_c', 'C_A_i'};
CSTR.OutputName = {'T', 'C_A'};
CSTR.StateName = {'C_A', 'T'};
CSTR.InputGroup.MV = 1;
CSTR.InputGroup.UD = 2;
CSTR.OutputGroup.MO = 1;
CSTR.OutputGroup.UO = 2;
CSTR
```

The first three lines specify labels for the input, output and state variables. The next four specify the signal type for each input and output. The designations MV, UD, MO, and UO mean *manipulated variable*, *unmeasured disturbance*, *measured output*, and *unmeasured output*. (See “Plant Inputs and Outputs” on page 2-3 for definitions.) For example, the code specifies that input 2 of model CSTR is an unmeasured disturbance. The last line causes the LTI object to be displayed, generating the following lines in the MATLAB® Command Window:

```
a =
               C_A         T
   C_A   -0.0285   -0.0014
   T     -0.0371   -0.1476

b =
              T_c     C_Ai
   C_A   -0.085   0.0238
   T     0.0802   0.4462

c =
          C_A    T
   T        0    1
   C_A      1    0


d =
          T_c  C_Ai
   T        0     0
   C_A      0     0

Input groups:
    Name    Channels
     MV        1
     UD        2

Output groups:
    Name    Channels
     MO        1
     UO        2

Continuous-time model
```

Input and Output Names

The optional `InputName` and `OutputName` properties affect the model displays, as in the above example. The toolbox also uses the `InputName` and `OutputName` properties to label plots and tables. In that context, the underscore character causes the next character to be displayed as a subscript.

Input and Output Types

General Case. As mentioned in “Overview” on page 2-2, Model Predictive Control Toolbox software supports three input types and two output types. In a Model Predictive Control Toolbox design, designation of the input and output types determines the controller dimensions and has other important consequences.

For example, suppose your plant structure were as follows.

Plant Inputs	Plant Outputs
Two manipulated variables (MVs)	Three measured outputs (MOs)
One measured disturbance (MD)	Two unmeasured outputs (UOs)
Two unmeasured disturbances (UDs)	

The resulting controller would have four inputs (the three MOs and the MD) and two outputs (the MVs). It would include feedforward compensation for the measured disturbance, and would assume that you wanted the unmeasured disturbances and outputs to be included as part of the regulator design.

If you didn’t want a particular signal to be treated as one of the above types, you could do one of the following:

- Eliminate the signal before using the model in controller design.
- For an output, designate it as unmeasured, then set its weight to zero (see “Output Weights” on page 3-20).
- For an input, designate it as an unmeasured disturbance, then define a custom state estimator that ignores the input (see “Disturbance Modeling and Estimation” on page 3-29).

Note By default, the toolbox assumes that unspecified plant inputs are manipulated variables, and unspecified outputs are measured. Thus, if you didn't specify signal types in the above example, the controller would have four inputs (assuming all plant outputs were measured) and five outputs (assuming all plant inputs were manipulated variables).

Example. For model `CSTR`, default Model Predictive Control Toolbox assumptions are incorrect. You must set its `InputGroup` and `OutputGroup` properties, as illustrated in the above code, or modify the default settings when you load the model into the design tool.

The toolbox provides a "helper" function called `setmpcsignals` to make type definition more convenient. For example

```
CSTR = setmpcsignals(CSTR, 'UD', 2, 'UO', 2);
```

sets `InputGroup` and `OutputGroup` to the same values as in the previous example. The `CSTR` display would then include the following lines:

```
Input groups:
       Name        Channels
    Unmeasured        2
    Manipulated       1



Output groups:
       Name       Channels
    Unmeasured       2
     Measured        1
```

Notice that `setmpcsignals` sets unspecified inputs to `Manipulated` and unspecified outputs to `Measured`.

See the Control System Toolbox documentation for additional information on LTI object properties.

Multiinput Multioutput (MIMO) Plants

Most Model Predictive Control Toolbox applications involve plants having multiple inputs and outputs. Model CSTR described in “State-Space Format” on page 2-5 is a MIMO plant, and the state space format extends naturally from single-input single-output (SISO) to MIMO plants.

You can also use the tf and zpk functions to build a MIMO plant model. For example, consider the following model of a distillation column [11], which has been used in many advanced control studies:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21.0s+1} & \frac{3.8e^{-8.1s}}{14.9s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} & \frac{4.9e^{-3.4s}}{13.2s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

Outputs y_1 and y_2 represent measured product purities. The control objective is to hold each at specified setpoints. To do so, the controller manipulates inputs u_1 and u_2, the flow rates of reflux and reboiler steam, respectively. Input u_3 is a measured feed flow rate disturbance.

The model consists of six transfer functions, one for each input/output pair. Each transfer function is the first-order-plus-delay form often used by process control engineers.

The following code shows how to define the distillation column model for use in the toolbox:

```
g11 = tf( 12.8, [16.7 1], 'IOdelay', 1.0);
g12 = tf(-18.9, [21.0 1], 'IOdelay', 3.0);
g13 = tf(  3.8, [14.9 1], 'IOdelay', 8.1);
g21 = tf(  6.6, [10.9 1], 'IOdelay', 7.0);
g22 = tf(-19.4, [14.4 1], 'IOdelay', 3.0);
g23 = tf(  4.9, [13.2 1], 'IOdelay', 3.4);
DC = [g11 g12 g13
      g21 g22 g23];
DC.InputName = {'Reflux Rate', 'Steam Rate', 'Feed Rate'};
DC.OutputName = {'Distillate Purity', 'Bottoms Purity'};
DC = setmpcsignals(DC, 'MD', 3)
```

The code defines the individual transfer functions, and then forms a matrix in which each row contains the transfer functions for a particular output, and each column corresponds to a particular input. The code also sets the signal names and designates the third input as a measured disturbance. The resulting LTI object display is as follows:

```
Transfer function from input "Reflux Rate" to output...
                                     12.8
 Distillate Purity:  exp(-1*s) * ----------
                                  16.7 s + 1

                                  6.6
 Bottoms Purity:  exp(-7*s) * ----------
                               10.9 s + 1

Transfer function from input "Steam Rate" to output...
                                    -18.9
 Distillate Purity:  exp(-3*s) * --------
                                  21 s + 1


                                 -19.4
 Bottoms Purity:  exp(-3*s) * ----------
                               14.4 s + 1

Transfer function from input "Feed Rate" to output...
                                       3.8
 Distillate Purity:  exp(-8.1*s) * ----------
                                    14.9 s + 1


                                    4.9
 Bottoms Purity:  exp(-3.4*s) * ----------
                                 13.2 s + 1
Input groups:
       Name        Channels
     Measured         3
    Manipulated      1,2

Output groups:
      Name      Channels
    Measured      1,2
```

LTI Model Characteristics

Control System Toolbox software provides functions for analyzing LTI models. Some of the more commonly used are listed below. Type the example code at the MATLAB® prompt to see how they work for the CSTR example.

Example	Intended Result
`dcgain(CSTR)`	Calculate gain matrix for the CSTR model's input/output pairs.
`impulse(CSTR)`	Graph CSTR model's unit-impulse response.
`ltiview(CSTR)`	Open the LTI Viewer with the CSTR model loaded. You can then display model characteristics by making menu selections.
`pole(CSTR)`	Calculate CSTR model's poles (to check stability, etc.).
`step(CSTR)`	Graph CSTR model's unit-step response.
`zero(CSTR)`	Compute CSTR model's transmission zeros.

System Identification Toolbox™ Models

System Identification Toolbox™ software for MATLAB® generates LTI models based on plant input/output data. This section explains how to use such models in Model Predictive Control Toolbox™ software:

- “System Identification Model Definition Example” on page 2-14
- “Converting a System Identification Toolbox™ Model to an LTI Object” on page 2-15
- “Step-Response Models” on page 2-17

Note The System Identification Toolbox product is optional. To determine whether your installation includes it, type `ver` at the MATLAB prompt, and look for “System Identification Toolbox” in the list of installed products.

System Identification Model Definition Example

To use System Identification Toolbox software, you first create an `iddata` object containing measured values of your plant input and output signals. The following example uses the System Identification Toolbox tutorial data set, a temperature-control application. Load the data as follows:

```
load dryer2
```

This creates vectors `u2` and `y2` in your workspace. Vector `u2` is a sequence of 1000 plant input values (electrical power), and `y2` is the corresponding output sequence (1000 temperature values). The sampling period is 0.08 second.

Create an `iddata` object called `dry` as follows:

```
dry = iddata(y2,u2,0.08);
```

Once the data has been loaded, use the System Identification Toolbox GUI or commands to determine a model that best fits the data. For example, the commands

```
dry.InputName = 'Power';
dry.OutputName = 'Temperature';
ze = detrend(dry(1:300));
m1 = pem(ze);
```

create a System Identification Toolbox model called `m1` (see the System Identification Toolbox documentation for a detailed explanation). If you type

```
whos m1
```

at the MATLAB prompt, the displayed result is:

```
Name      Size                        Bytes  Class

  m1        4-D                         22470  idss object
```

Notice that `m1` is an `idss` object, one of seven possible System Identification Toolbox model types (`idgrey`, `idarx`, `idpoly`, `idproc`, `idss`, `idmodel`, and `idfrd`). The `pem` function settings govern the type of model generated.

Like other System Identification Toolbox objects, `m1` defines a model structure and adjustable parameter values that best fit the data. It also contains toolbox-specific information, such as the algorithm used to estimate the parameters.

Converting a System Identification Toolbox™ Model to an LTI Object

After you've created a System Identification Toolbox model based on your data, you must convert it to a standard LTI object before using it in the Model Predictive Control Toolbox software. System Identification Toolbox software provides a special conversion function (`ss`).

Note An exception is the `mpc` function described in "Controller Definition" on page 4-2, which can use System Identification Toolbox models directly.

Creating an LTI State-Space Model

Use the `ss` function to convert a System Identification Toolbox model object (except the `idfrd` type) to a standard Control System Toolbox `ss` (state-space) object, which is the form used internally by the Model Predictive Control Toolbox software. For example,

```
m1ss = ss(m1)
```

converts `m1`, a System Identification Toolbox object, to `m1ss`, an LTI `ss` object, and displays the following:

```
a =
               x1        x2        x3
   x1     0.9492   -0.2127   0.03679
   x2     0.2599    0.6523    0.2342
   x3   -0.04822   -0.6639    0.1393


b =
                 Power  v@Temperatur
   x1    -0.0005008      0.002613
   x2      -0.01335    -0.0002421
   x3      -0.06729     -0.004463

c =
                        x1        x2        x3
   Temperature       14.09    -0.108  -0.08164

d =
                         Power  v@Temperatur
   Temperature               0       0.03968

Input groups:
      Name      Channels
    Measured       1
     Noise         2

Sampling time: 0.08
Discrete-time model
```

Here `m1ss` is a third-order, discrete-time, state-space model with a sampling time of 0.08, one output (`Temperature`), and two inputs (`Power` and `v@Temperatur`).

Noise Inputs

System Identification Toolbox software automatically creates a noise input for each output to model the impact of unmeasured disturbances and measurement noise. In the above example, there is one output (`Temperature`). Its associated noise input is `v@Temperatur`.

System Identification Toolbox software designates the noise inputs using the LTI model's `InputGroup` property. In the above example, channel 2 (`v@Temperatur`) is classified as `Noise`, while channel 1 (`Power`) is `Measured`.

When you use such a model in Model Predictive Control Toolbox software, `Noise` inputs will be treated as unmeasured disturbances, and `Measured` inputs will be treated as manipulated variables. (See "Overview" on page 2-2 for discussion of input types.) If these defaults are inappropriate, you must correct them prior to using the model in a design. If you are using the design tool, "Loading a Plant Model" on page 3-3 shows how to specify signal types. When using commands, you need to set the models `InputGroup` and `OutputGroup` properties, as illustrated in "LTI Object Properties" on page 2-7.

Note As discussed in "Disturbance Modeling and Estimation" on page 3-29, unmeasured disturbance inputs influence the default controller design. A System Identification Toolbox model of its `Noise` inputs might fit the given data, but another experiment might yield a very different noise model. If a controller designed using such a model seems to work well during setpoint changes but is slow to eliminate disturbances or exhibits steady-state error, try modifying the Model Predictive Control Toolbox disturbance modeling settings.

Step-Response Models

Early predictive control implementations used finite step-response or finite impulse-response models, often called *nonparametric* LTI models (see [6] for example). Such models are easy to determine from plant data ([3], [7]) and they have intuitive appeal.

System Identification Toolbox software includes tools for nonparametric model identification. See the System Identification Toolbox documentation for details.

For example, given the `iddata` object `ze` (defined in "System Identification Model Definition Example" on page 2-14), you could type:

```
m2 = step(ze);
```

This identifies a finite step-response model, m2, which is a System Identification Toolbox idarx object. Then typing

```
step(m2)
```

would display its unit-step response.

You could convert m2 to an LTI object for use in the Model Predictive Control Toolbox design (see “Creating an LTI State-Space Model” on page 2-15). The disadvantage is that the model will be high-order, especially if your plant is MIMO. For example, converting m2 generates an ss object of order 69.

High-order models can degrade certain Model Predictive Control Toolbox operations, such as estimator design. As is the case with most recent predictive control implementations (see [5] for example), Model Predictive Control Toolbox algorithms work best with a low-order parametric model. For example, reference [10] describes a systematic approach that identifies a step-response model as an intermediate step. The System Identification Toolbox documentation also advocates this approach.

Using Simulink® to Develop LTI Models

LTI and System Identification Toolbox™ models discussed in the previous sections are *linear* dynamic models. Most real systems are nonlinear. If you'd like to simulate Model Predictive Control Toolbox™ control of a nonlinear system, you must model the plant in Simulink®.

Although a Model Predictive Control Toolbox controller can regulate a nonlinear plant, the model used within the controller must be linear. In other words, the controller employs a linear approximation of the nonlinear plant. The accuracy of this approximation is a key issue affecting controller performance.

This raises the question: how to obtain the approximation? The usual approach is to *linearize* the nonlinear plant at a specified *operating point*. The Simulink environment provides two ways to accomplish this:

- "Linearization Using Simulink® Control Design™" on page 2-19
- "Linearization Using Simulink® Functions" on page 2-24

Linearization Using Simulink® Control Design™

Simulink® Control Design™ software is an optional product that supports model linearization. To determine whether or not your license includes it, type `ver` at the MATLAB® prompt, and note whether or not "Simulink Control Design" appears in the resulting product list.

The Simulink Control Design documentation includes extensive background on linearization and several examples. You can linearize a Simulink model of the plant alone, or a model that includes both the plant and its controller.

Figure 2-3 is a Simulink model of a continuous stirred-tank reactor (CSTR). It is similar to the one in Figure 2-2, except that here the model is nonlinear and includes an additional input: the feed temperature. This code is stored in the Model Predictive Control Toolbox `demo` folder. You can open it at the MATLAB prompt by typing:

```
CSTR_OL
```

As shown in Figure 2-3, the three inputs are being held at constant values: 10 kmol/m^3 for the feed concentration, and 298.15 K for the feed and coolant temperatures. Like the model of Figure 2-2, there are two state variables: the reactor temperature and the reactant concentration leaving the reactor. The Simulink model defines their initial conditions to be 311.27 K and 8.57 kmol/m^3, which are at steady state (or equlibrium condition) for the given inputs. (If you run the simulation, the outputs will stay at their initial conditions.)

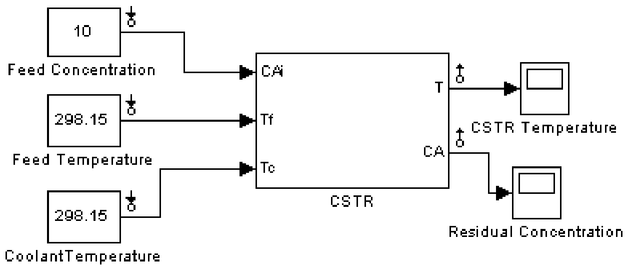


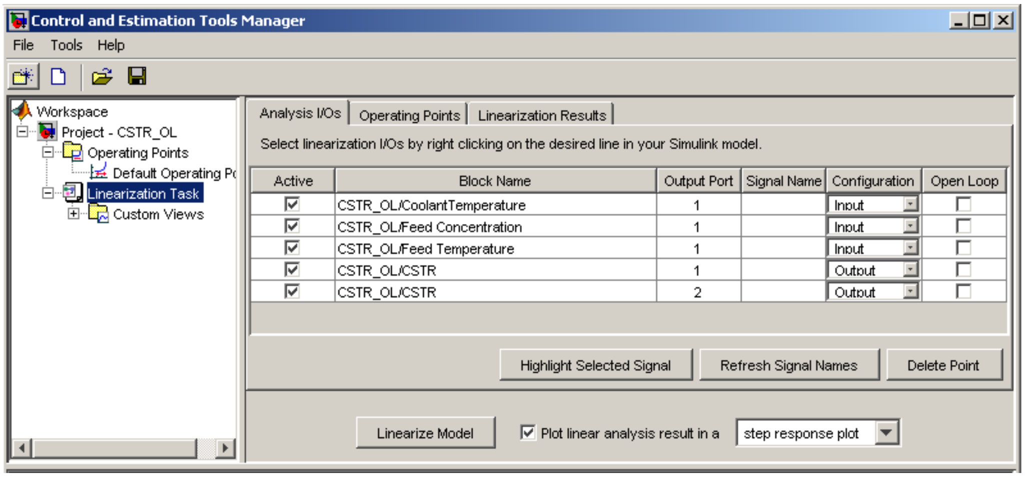
Figure 2-3: Simulink® Model of a Nonlinear Chemical Reactor

Note "Nonlinear Plants" on page 3-50 shows how to linearize this model when a Model Predictive Control block is included.

To linearize Figure 2-3, first designate the input and output signals to be retained in the linear approximation. In general, you would choose signals that will be connected to a controller. In Figure 2-3, all the signals have been selected by adding *linearization points*, i.e., by right-clicking a signal and selecting either **Input Point** or **Output Point** from the **Linearization Points** submenu.

Next, create a linearization project within the Simulink Control and Estimation Tools Manager. From the **Tools** menu of the Simulink model, select **Control Design/Linear Analysis**.

Figure 2-4 shows the resulting window for the CSTR example. The tool automatically defines the **Default Operating Point** entry, in this case the initial steady-state condition.



Select linearization I/Os by right clicking on the desired line in your Simulink model.

Active	Block Name	Output Port	Signal Name	Configuration	Open Loop
✓	CSTR_OL/CoolantTemperature	1		Input	☐
✓	CSTR_OL/Feed Concentration	1		Input	☐
✓	CSTR_OL/Feed Temperature	1		Input	☐
✓	CSTR_OL/CSTR	1		Output	☐
✓	CSTR_OL/CSTR	2		Output	☐

✓ Plot linear analysis result in a step response plot

Figure 2-4: CSTR Model Linearization Using Simulink® Control Design

Linearization at a Specified Operating Point

To calculate the linear approximation at a particular operating point, select **Linearization Task** in the tree (highlighting it as shown in Figure 2-4), select the desired operating point on the **Operating Points** tab (if more than one have been defined), and then click the **Linearize Model** button.

Figure 2-4 shows the tool's state after a linearized model has been created at the default operating point. This model appears in the tree as the icon labeled **Model**. The tree has been expanded to show its associated operating point, which in this case is labeled **Default Operating Point**. You can select the model in the tree, right-click, and then export it to MATLAB, making it available for use in another tool.

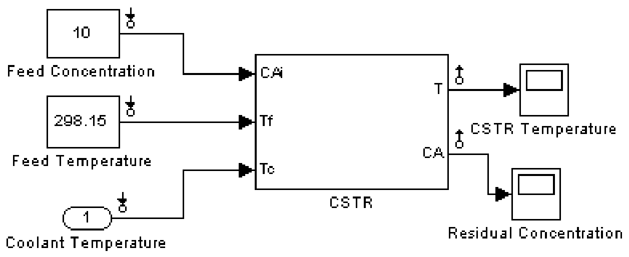
Determining a New Operating Point

You are likely to need to modify a nonlinear model's operating point. For example, the above `CSTR` model has a poor reactant conversion. The feed contains 10 kmol/m^3, and the residual is 8.57 kmol/m^3, so only 1.43 has reacted (14.3% conversion).

Suppose that you'd like to react 80% (i.e., a residual concentration of 2 kmol/m^3). To increase the conversion you need to increase the CSTR temperature, but how much? Also, to change the CSTR temperature you need to change the coolant temperature, but how much?

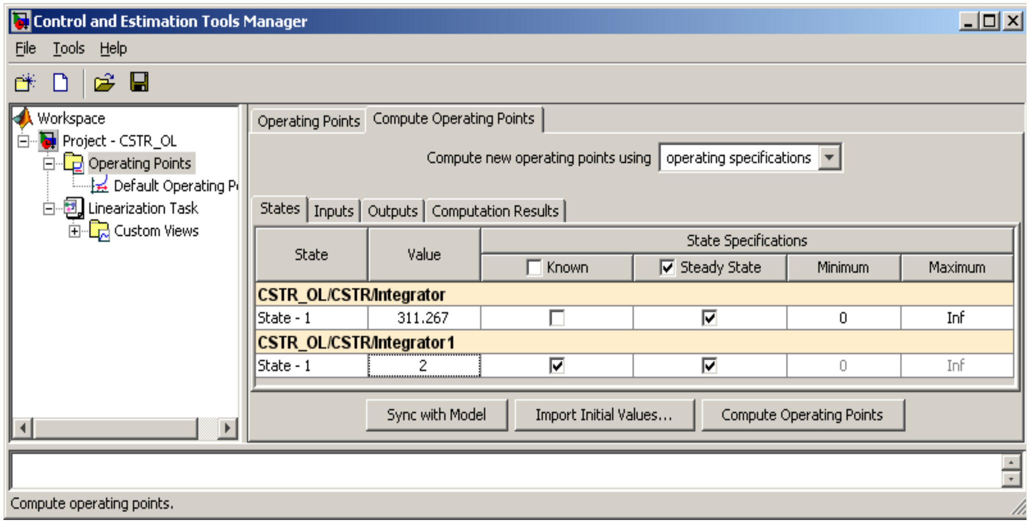
One approach would be to change the coolant temperature, run a simulation of sufficient duration to reach a new steady state, check the final residual concentration, and repeat until you achieve the desired 2.0 kmol/m^3 residual. This is tedious and essentially impossible in a more complex situation where you are trying to match several targets simultaneously.

Simulink Control Design software can search for a new steady state operating point that achieves the desired conversion. First, you must modify the model so Simulink can change the coolant temperature. One way is to represent the coolant temperature with an Inport block, as shown below (compare to Figure 2-3, which uses a Constant block.



Save this modified model under a new name. Then from the **Tools** menu, select **Control Design/Linear Analysis** as before. If the Control and Estimation Tools Manager window containing `CSTR_OL` is still open (as shown in Figure 2-4), a new linearization project will be inserted. Otherwise, the window will open with `CSTR_OL` as a new project.

It will contain a default operating point. This is as before except that the coolant temperature appears as an input and defaults to zero. To modify this, select **Operating Points** in the tree, and select the **Compute Operating Points** tab. On this pane, click the **States** tab and set the check boxes as shown below.



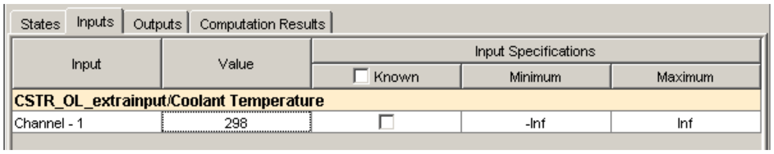
Compute new operating points using: operating specifications

State	Value	State Specifications			
		☐ Known	☑ Steady State	Minimum	Maximum
CSTR_OL/CSTR/Integrator					
State - 1	311.267	☐	☑	0	Inf
CSTR_OL/CSTR/Integrator1					
State - 1	2	☑	☑	0	Inf

Compute operating points.

Also set the desired value of the second state (the residual concentration) to 2, as shown. You are asking for a new operating point in which one state is specified (known) and both are at steady state. The reactor temperature **value** (311.267) is an initial guess. The tool will search for a value that satisfies all the specifications.

Next, click the **Inputs** tab and verify that the coolant temperature input has its **Known** check box unselected as shown below.



Input	Value	Input Specifications		
		☐ Known	Minimum	Maximum
CSTR_OL_extrainput/Coolant Temperature				
Channel - 1	298	☐	-Inf	Inf

The **value** is an initial guess that will be changed. You can set it to 298, as shown above, to help the tool converge its trial-and-error calculations. (The default guess of 0 should also work here, but it is good practice to supply a problem-specific guess to aid convergence.)

Finally, click the **Compute Operating Point** button. A calculation progress pane shows the specification error at each iteration. When it's finished, you should see the line, "Operating point specifications were successfully met" and a new operating point should appear in the tree. Click this and observe that the required reactor temperature is 373.13 K, and the required coolant temperature is 305.20 K.

Let's calculate a new linearized model at this condition, comparing it to that obtained at the original point. Suppose you exported the original model as `Model1`, and the other as `Model2`. The following command would compare their step responses:

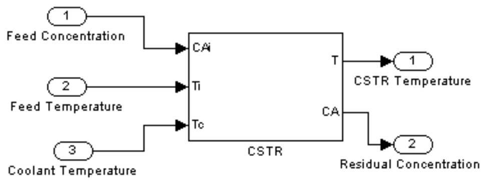
```
step(Model1, Model2)
```

You should see some significant quantitative and qualitative differences, especially in the response to a change in feed concentration. At the original low-conversion state, increasing the feed concentration increases both the reactor temperature and the residual concentration. At high conversion, however, the reaction is more sensitive to temperature changes. Increasing the feed concentration causes an initial rise in the residual concentration, but the increased temperature accelerates the reaction rate and the residual concentration goes *below* its initial value. Thus, if one were trying to control conversion by adjusting the feed concentration, a model-based controller designed for low conversion would be certain to fail at high conversion.

Linearization Using Simulink® Functions

Another approach is to linearize the model using standard Simulink functions. This is more restrictive: you cannot perform open loop analysis of the Simulink model, and the signals to be retained in the linearized model must be connected to an inport or outport block. On the other hand, Simulink Control Design software is not needed.

The following diagram shows a modification of Figure 2-3 with three inport blocks designating the input signals (on the left side of the model) and two outports designating the outputs (on the right).



Suppose this model were named `CSTR_INOUT`. The `linmod` command linearizes it as follows:

```
[a,b,c,d]=linmod('CSTR_INOUT')

a =

   -0.2505    1.9897
   -0.0880   -1.1669


b =

         0    1.0000    0.3000
    1.0000         0         0


c =

    1.0000         0
         0    1.0000


d =

     0     0     0
     0     0     0
```

By default, `linmod` uses the initial conditions defined in the model as the operating point. Options allow you to specify an operating point. The command outputs are the standard state-space matrices defining an LTI model. You can use these to create an LTI model as follows:

```
cstr = ss(a,b,c,d)
```

Bibliography

[1] Allgower, F., and A. Zheng, *Nonlinear Model Predictive Control*, Springer-Verlag, 2000.

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[4] Kouvaritakis, B., and M. Cannon, *Non-Linear Predictive Control: Theory & Practice*, IEE Publishing, 2001.

[5] Maciejowski, J. M., *Predictive Control with Constraints*, Pearson Education POD, 2002.

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[10] Wang, L., P. Gawthrop, C. Chessari, T. Podsiadly, and A. Giles, “Indirect approach to continuous time system identification of food extruder,” *J. Process Control*, Vol. 14, Number 6, pp. 603-615, 2004.

[11] Wood, R. K., and M. W. Berry, *Chem. Eng. Sci.*, Vol. 28, pp. 1707, 1973.

Designing the Controller Using the Design Tool GUI

Introduction

The Model Predictive Control Toolbox™ design tool is a graphical user interface for controller design. This GUI is part of the Control and Estimation Tools Manager GUI. To learn about using the Control and Estimation Tools Manager for linearization, see "Linearization Using Simulink® Control Design™" on page 2-19.

This section covers the following topics:

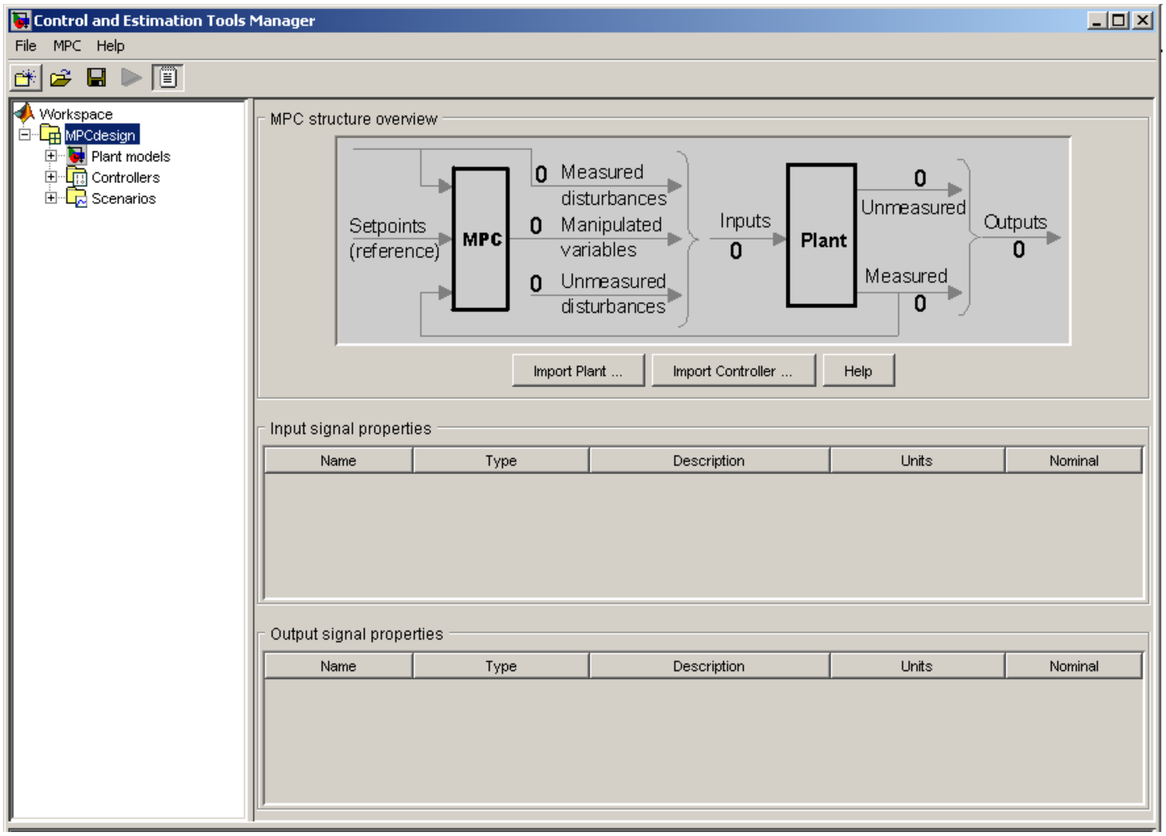
- "Starting the Design Tool" on page 3-2
- "Loading a Plant Model" on page 3-3
- "Signal Property Specifications" on page 3-5

Starting the Design Tool

Start the design tool by typing the MATLAB® command

```
mpctool
```

The Control and Estimation Tools Manager window appears, as shown below. By default, it contains a Model Predictive Control Toolbox task called **MPCdesign** (listed in the tree view on the left side of the window), which is selected, causing the view shown on the right to appear.



Input signal properties

Name	Type	Description	Units	Nominal

Output signal properties

Name	Type	Description	Units	Nominal

Figure 3-1: Model Predictive Control Toolbox™ Design Tool Initial View

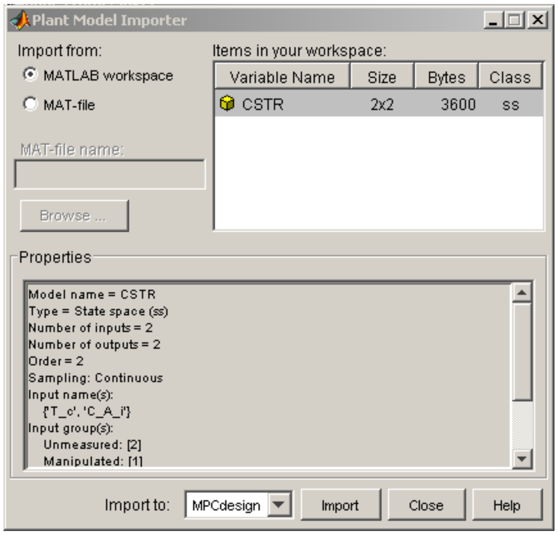
Loading a Plant Model

The first step in the design is to load a plant model. Its dimensions and signal characteristics set the context for the remaining steps. You can either load the model directly, as described in this section, or indirectly by importing a controller or a saved design (see “Loading Your Saved Work” on page 3-64).

The following example uses the `CSTR` model described in “State-Space Format” on page 2-5. Verify that the LTI object `CSTR` is in your MATLAB workspace (if necessary, create the model as explained in “State-Space Format” on page 2-5, and set its label and signal type properties as explained in “LTI Object Properties” on page 2-7).

Plant Model Importer Dialog Box

Click the **Import Plant** button in the design tool’s initial view (see Figure 3-1). The Plant Model Importer dialog box appears.



Variable Name	Size	Bytes	Class
CSTR	2x2	3600	ss

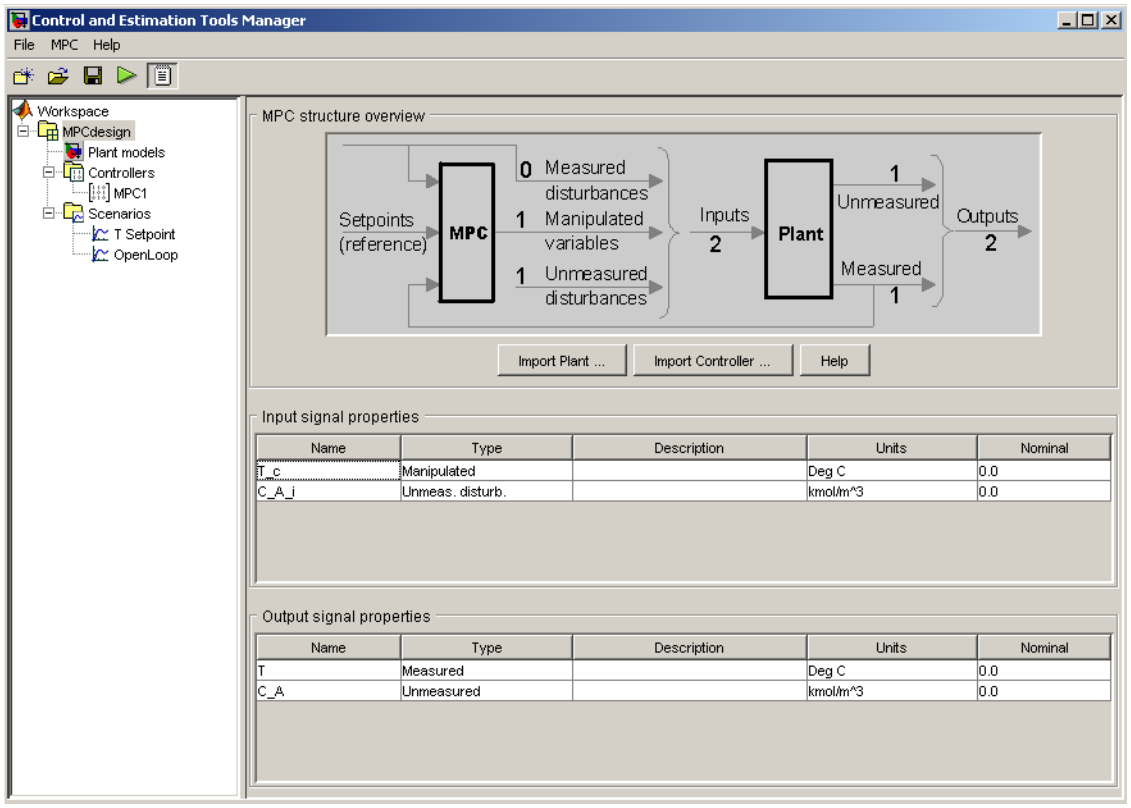
```
Model name = CSTR
Type = State space (ss)
Number of inputs = 2
Number of outputs = 2
Order = 2
Sampling: Continuous
Input name(s):
   {'T_c', 'C_A_i'}
Input group(s):
   Unmeasured: [2]
   Manipulated: [1]
```

Figure 3-2: Plant Model Importer Dialog Box

The **Import from MATLAB workspace** option button should be selected by default, as shown. The **Items in your workspace** table lists your LTI models. If `CSTR` doesn’t appear, define it as discussed in “State-Space Format” on page 2-5, then reopen this dialog box.

After `CSTR` appears, select it. The **Properties** list displays the number of inputs and outputs, their names and signal types, etc.

Click the **Import** button. This loads `CSTR` into the design tool. Then click the **Close** button (otherwise the dialog box remains visible in case you want to import another model). The design tool should appear as in Figure 3-3.



Input signal properties

Name	Type	Description	Units	Nominal
T_c	Manipulated		Deg C	0.0
C_A_i	Unmeas. disturb.		kmol/m^3	0.0

Output signal properties

Name	Type	Description	Units	Nominal
T	Measured		Deg C	0.0
C_A	Unmeasured		kmol/m^3	0.0

Figure 3-3: Model Predictive Control Toolbox™ Design Tool's Signal Definition View

Signal Property Specifications

The figure's graphical display indicates that you've imported a plant model by showing the number of inputs and outputs, and the number in each subclass: measured disturbance, manipulated variables, etc.). Also, the tables labeled **Input signal properties** and **Output signal properties** fill with data:

- The **Name** entries are from the `CSTR` model's `InputName` and `OutputName` properties (the design tool assigns defaults if necessary). You can edit these at any time.

- The **Type** entries are from the `CSTR` model's `InputGroup` and `OutputGroup` properties. (The design tool defaults all unspecified inputs to manipulated variables and all unspecified outputs to measured.)

Note Once you leave this view, if you subsequently change a signal type, you will have to restart the design. Be sure the signal types are correct at the beginning.

- The **Description** and **Unit** entries are optional. You can enter the values shown in Figure 3-3 manually. As you will see, the design tool uses them to label plots and other tables.
- The **Nominal** entries are initial conditions for simulations. The design tool default is `0.0`.

Navigation Using the Tree View

The *tree* in the left-hand frame of Figure 3-3 shows that the default **MPCdesign** *node* (see Figure 3-1) has been renamed **CSTRcontrol** by clicking the name, waiting for the usual edit box to appear, typing the new name, and pressing **Enter** to finalize the choice.

Figure 3-3 also shows three new nodes below **CSTRcontrol**. These activate once you've imported a plant model (or controller).

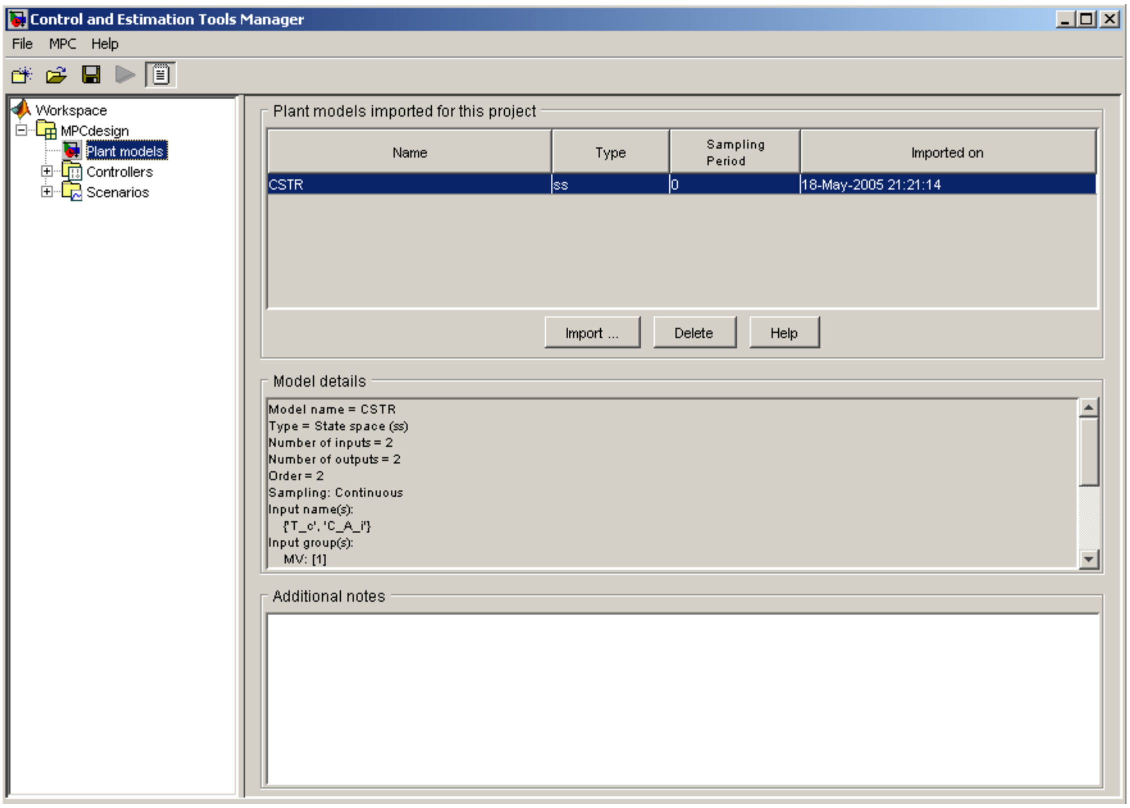
In general, clicking a node displays a *view* supporting a particular design activity.

Project View (Signal Properties Tables)

For example, clicking the *project* node (**CSTRcontrol** in Figure 3-3) allows you to review and edit the signal properties tables.

Listing Your Plant Models

Select the **Plant models** node to list the plant models you've imported, as shown in Figure 3-4. (Each model name is editable.) The **Model details** section displays properties of the selected model. There is also a space to enter notes describing the model's special features. Buttons allow you to import a new model or delete one you no longer need.



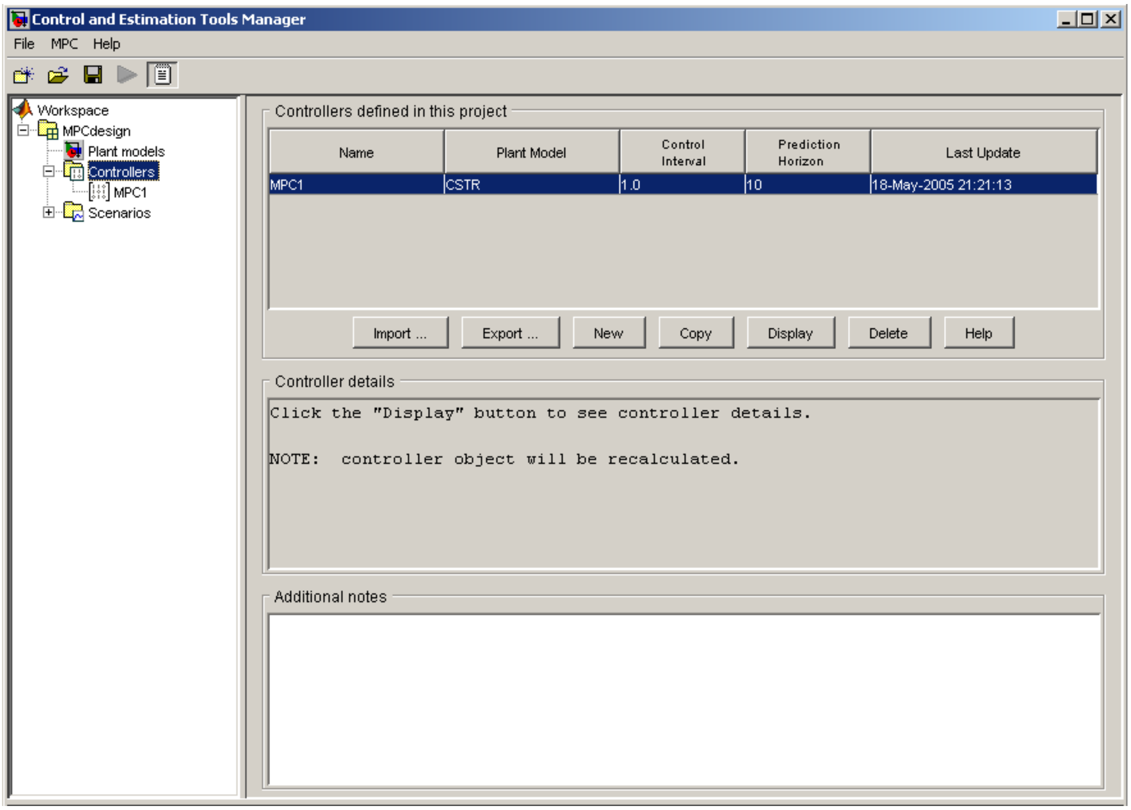
Name	Type	Sampling Period	Imported on
CSTR	ss	0	18-May-2005 21:21:14

```
Model name = CSTR
Type = State space (ss)
Number of inputs = 2
Number of outputs = 2
Order = 2
Sampling: Continuous
Input name(s):
   {'T_c', 'C_A_i'}
Input group(s):
   MV: [1]
```

Figure 3-4: Plant Models View with CSTR Model Selected

Viewing Your Controllers

Next, select **Controllers**. The view shown in Figure 3-5 appears. A **+** sign to the left of **Controllers** indicates that it contains subnodes. You can click a **+** sign to expand the tree (as shown in Figure 3-5, where the **+** sign has changed to a **–** sign).



Name	Plant Model	Control Interval	Prediction Horizon	Last Update
MPC1	CSTR	1.0	10	18-May-2005 21:21:13

```
Click the "Display" button to see controller details.

NOTE:  controller object will be recalculated.
```

Figure 3-5: Controllers View

The table at the top of Figure 3-5 lists all the controllers you've defined. The design tool automatically creates a controller containing Model Predictive Control Toolbox defaults, naming it **MPC1**. It is a subnode of **Controllers**.

Note If you define additional controllers, they will appear here. For example, you might want to test several options, saving each as a separate controller, making it easy to switch from one to another during testing.

The table **Controllers defined in this project** allows you to edit the controller name and gives you quick access to three important design parameters: **Plant Model**, **Control Interval**, and **Prediction Horizon**. All are editable, but leave them at their default values for this example.

The buttons shown in Figure 3-5 let you do the following:

- Import a controller designed previously and stored either in your workspace or in a MAT-file.
- Export the selected controller to your workspace.
- Create a new controller initialized to Model Predictive Control Toolbox defaults.
- Copy the selected controller, creating a duplicate you can modify.
- Delete the selected controller.

You can also right-click the **Controllers** node to access menu options **New**, **Import**, and **Export**, or one of its subnodes to access menu options **Copy**, **Rename**, **Export**, and **Delete**.

Select the **MPC1** node to display Model Predictive Control Toolbox default controller settings. ("Changing Controller Settings" on page 3-17 covers this view in detail).

Viewing Simulation Scenarios

A *scenario* is a set of conditions defining a simulation. The design tool creates a default scenario and names it **Scenario1**. To view it, click the **+** symbol next to the **Scenarios** node, and select the **Scenario1** subnode. You should see a view like that shown in Figure 3-6.

Whenever you select the **Scenarios** node, you see a table summarizing your current scenarios (not shown). Its function is similar to the **Controllers** view described previously.

Linear Simulations

You will usually want to test your controller in simulations. The Model Predictive Control Toolbox™ design tool makes it easy to run closed-loop simulations involving a Model Predictive Control Toolbox controller and an LTI plant model. This plant can differ from that used in the controller design, allowing you to test your controller's sensitivity to prediction errors (see "Robustness Testing" on page 3-40).

This section covers the following topics:

- "Defining Simulation Conditions" on page 3-10
- "Running a Simulation" on page 3-11
- "Open-Loop Simulations" on page 3-14

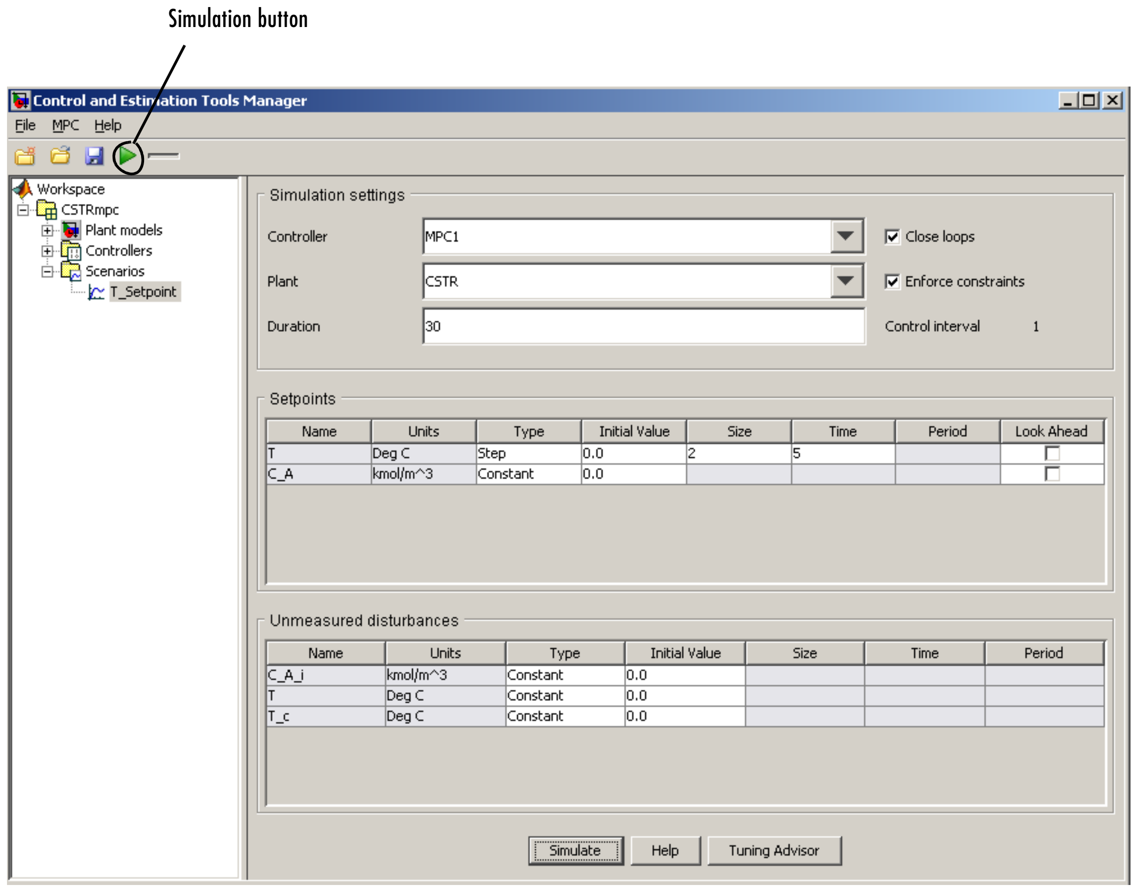
Defining Simulation Conditions

To define simulation conditions, select an existing scenario node or create a new one, and then edit its tabular fields to define your conditions.

Figure 3-6 shows the result of renaming the default **Scenario1** node to **T Setpoint** and editing its default conditions. The required editing steps are as follows:

- Increase **Duration** from `10` to `30`.
- Locate the tabular data defining the reactor temperature setpoint (first row of the upper table in Figure 3-6).
  - Click the **Type** table cell and select `Step` from the list of choices.
  - Change **Size** from `1.0` to `2`.
  - Change **Time** from `1.0` to `5`.

Note The **Control Interval** is a property of the controller being used (`MPC1` in this case). To change it, select the controller node in the tree, and then edit the value on the **Model and Horizons** tab (see "Model and Horizons" on page 3-17). Such a change would apply to all simulations involving that controller.



Simulation settings

Controller: MPC1 — [✓] Close loops
Plant: CSTR — [✓] Enforce constraints
Duration: 30 — Control interval 1

Setpoints

Name	Units	Type	Initial Value	Size	Time	Period	Look Ahead
T	Deg C	Step	0.0	2	5		☐
C_A	kmol/m^3	Constant	0.0				☐

Unmeasured disturbances

Name	Units	Type	Initial Value	Size	Time	Period
C_A_i	kmol/m^3	Constant	0.0			
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	0.0			

Figure 3-6: CSTR Temperature Setpoint Change Scenario

Running a Simulation

To run a simulation, do *one* of the following:

- Select the scenario you want to run, and click its **Simulate** button (see the bottom of Figure 3-6).
- Click the toolbar's **Simulation** button, which is the triangular icon shown on the top left of Figure 3-6.

The toolbar method runs the *current scenario*, i.e., the one most recently selected or modified.

Try running the **T Setpoint** scenario. This should generate the two response plot windows shown in Figure 3-7 and Figure 3-8.

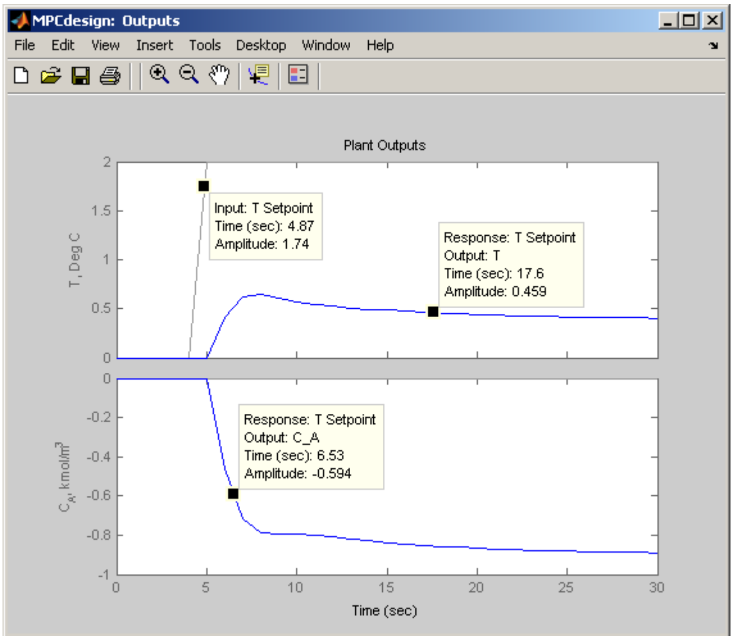


Figure 3-7: Plant Outputs for T Setpoint Scenario with Added Data Markers

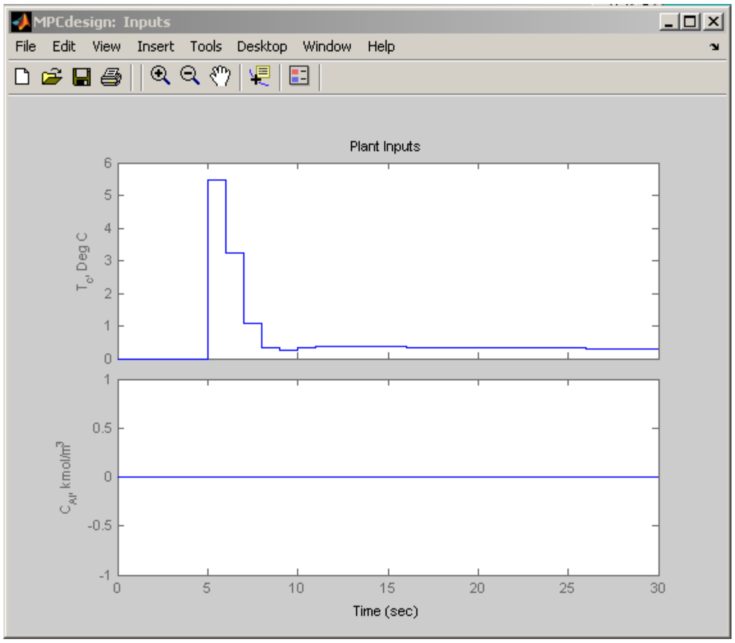


Figure 3-8: Plant Inputs for the T Setpoint Scenario

Figure 3-7 shows that the reactor temperature setpoint increases suddenly by 2 degrees at t = 5, as you specified when defining the scenario in “Defining Simulation Conditions” on page 3-10. Unfortunately, the temperature does not track the setpoint very well, and there is a persistent error of about 1.6 degrees at the end of the simulation.

Also, the controller requests a sudden jump in the coolant temperature (see the upper graph in Figure 3-8), which might be difficult to deliver in practice. (The lower graph in Figure 3-8 shows that the feed concentration, C_{Ai}, remains constant, as specified in the scenario.)

See “Changing Controller Settings” on page 3-17 for ways to overcome these deficiencies.

Note Figure 3-7 has *data markers*. To add these, left-click the curve to create the data marker. Drag a marker to relocate it. Left-click in a graph's white space to erase its markers. For more information on data markers, see the Control System Toolbox™ documentation.

Open-Loop Simulations

By default, scenarios are *closed loop*, i.e., an active controller adjusts the manipulated variables entering your plant based on feedback from the plant outputs. You can also run *open-loop* simulations that test the plant model without feedback control.

For example, you might want to check your plant model's response to a particular input without opening another tool. You might also want to display unmeasured disturbance signals before using them in a closed-loop simulation.

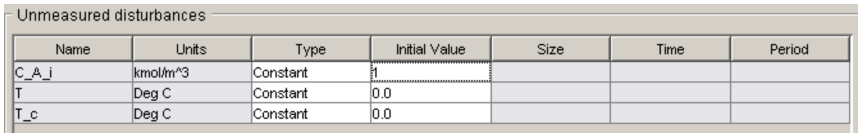
To see how this works, create a new scenario by right-clicking the **T Setpoint** node in the tree, and selecting **Copy Scenario** in the resulting menu. Rename the copy **OpenLoop**.

Select **OpenLoop** in the tree. On its scenario view, change **Duration** to `100`, and turn off (clear) **Close loops**.

Open-loop simulations ignore the **Setpoints** table settings, so there's no need to modify them.

If `CSTR` had a measured disturbance input, the pane would contain another table allowing you to specify it.

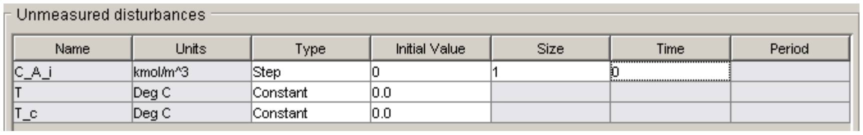
For this example, focus on the **Unmeasured disturbances** table. Configure it as shown below.



Unmeasured disturbances

Name	Units	Type	Initial Value	Size	Time	Period
C_A_i	kmol/m^3	Constant	1			
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	0.0			

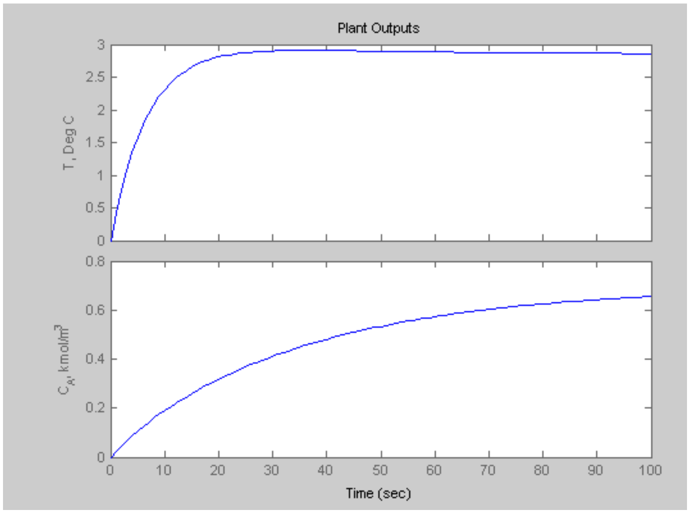
The `C_A_i` input's nominal value is `0.0` (see Figure 3-3), so the above models a sudden increase to `1` at the beginning of the simulation. The following is an equivalent setup using the `Step` type.



Unmeasured disturbances

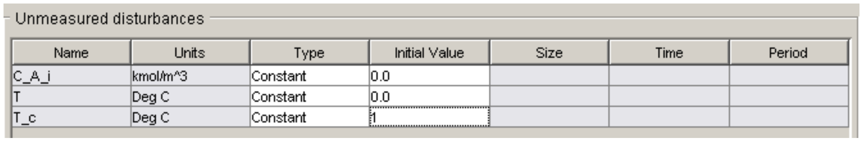
Name	Units	Type	Initial Value	Size	Time	Period
C_A_i	kmol/m^3	Step	0	1	0	
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	0.0			

Using one of these, simulate the scenario (click its **Simulate** button). The output response plot should be as shown below.



This is the `CSTR` model's open-loop response to a unit step in the C_{Ai} disturbance input.

You could also set up the table as shown below.

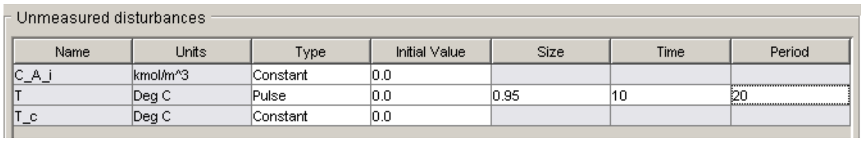


Unmeasured disturbances

Name	Units	Type	Initial Value	Size	Time	Period
C_A_i	kmol/m^3	Constant	0.0			
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	1			

This simulation would display the open-loop response to a unit step in the T_c manipulated variable input (try it).

Finally, set it up as follows.



Unmeasured disturbances

Name	Units	Type	Initial Value	Size	Time	Period
C_A_i	kmol/m^3	Constant	0.0			
T	Deg C	Pulse	0.0	0.95	10	20
T_c	Deg C	Constant	0.0			

This adds a pulse to the `T` output. The pulse begins at time $t = 10$, and lasts 20 time units. Its height is 0.95 degrees.

Run the simulation. The output response plot displays the pulse (see Figure 3-9). In this case, the `T` output's nominal value is zero, so you only see the pulse. (If the `T` output had a nonzero nominal value, the pulse would add to that.)

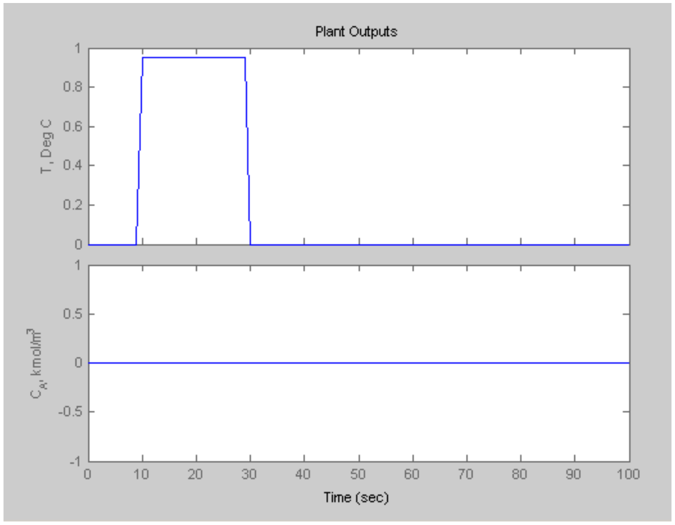


Figure 3-9: Response Plot Showing Open-Loop Pulse Disturbance

If you were to run a *closed-loop* simulation with this same `T` disturbance, the controller would attempt to hold `T` at its setpoint, and the result would differ from that shown in Figure 3-9.

Changing Controller Settings

The simulations shown in Figure 3-7 and Figure 3-8 used the default controller settings. These often work well, but the `CSTR` application is an exception. The following discussion covers the main controller options in more detail, and shows how to tune a controller for better performance.

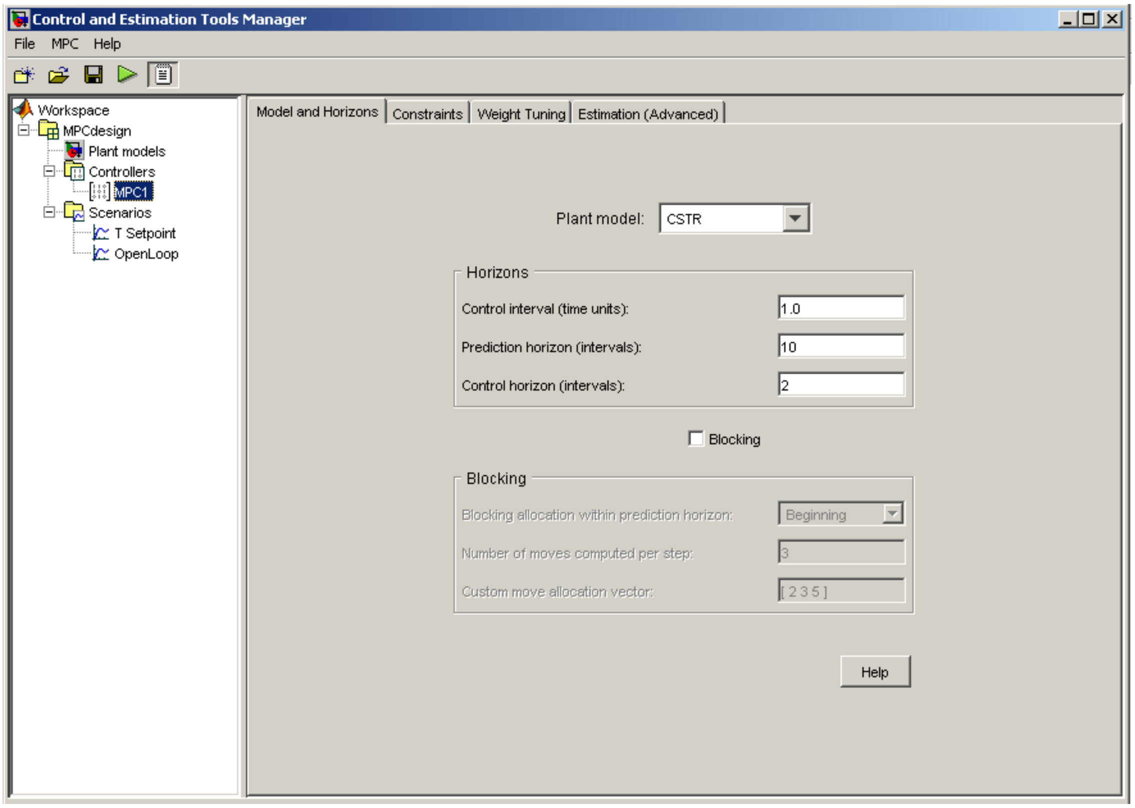
This section covers the following topics:

Model and Horizons

Select your **MPC1** controller in the tree. The view shown in Figure 3-10 should appear. If necessary, click the **Model and Horizons** tab to bring it to the front. This tab contains the following controller options:

- **Plant model** specifies the LTI model to be used for controller predictions.
- **Control interval** sets the elapsed time between successive adjustments of the controller's manipulated variables.
- **Prediction horizon** is the number of control intervals over which the outputs are to be optimized.
- **Control horizon** sets the number of control intervals over which the manipulated variables are to be optimized.
- Selecting the **Blocking** option gives you more control over the way in which the controller's *moves* are allocated.

Leave all of the **Model and Horizons** tab settings at their default values for now.



Plant model: CSTR

Horizons

Control interval (time units): 1.0

Prediction horizon (intervals): 10

Control horizon (intervals): 2

☐ Blocking

Blocking

Blocking allocation within prediction horizon: Beginning

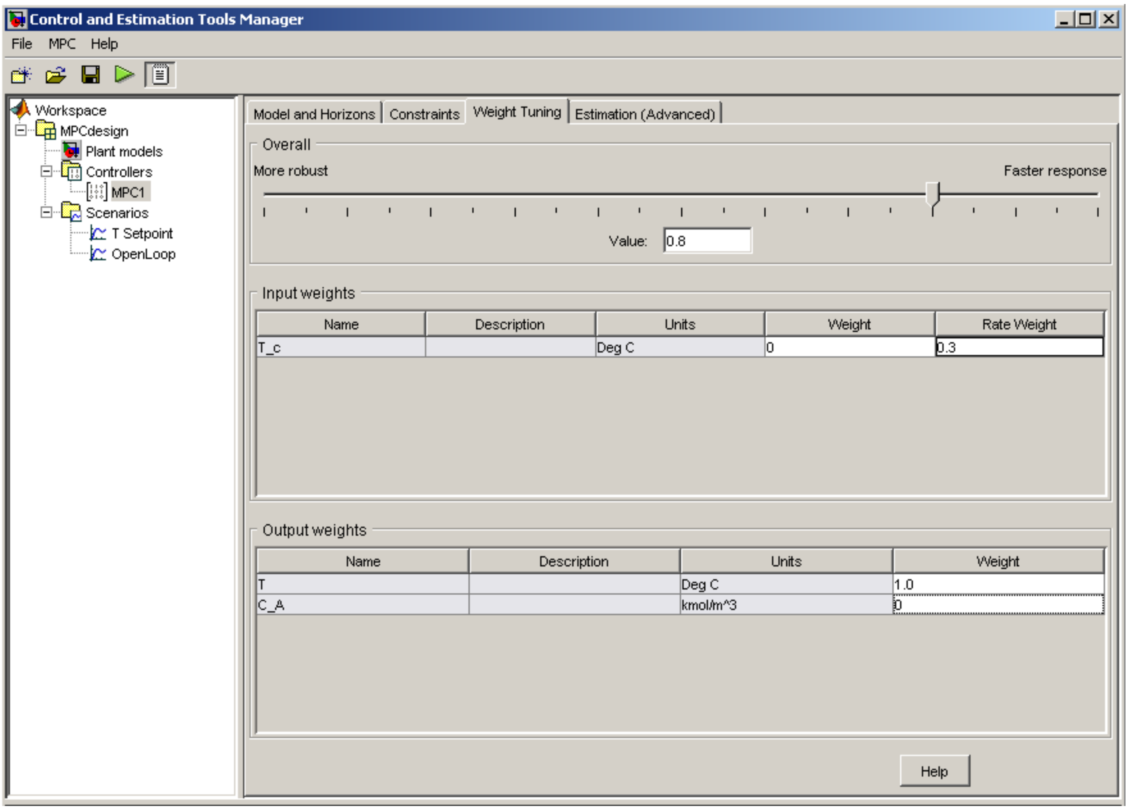
Number of moves computed per step: 3

Custom move allocation vector: [2 3 5]

Figure 3-10: Controller Options — Model and Horizons Tab

Weight Tuning

Click the **Weight Tuning** tab. The view shown in Figure 3-11 appears.



Value: 0.8

Input weights

Name	Description	Units	Weight	Rate Weight
T_c		Deg C	0	0.3

Output weights

Name	Description	Units	Weight
T		Deg C	1.0
C_A		kmol/m^3	0

Figure 3-11: Controller Options — Weight Tuning Tab

Make the following changes (already done in the above view):

- In the **Input weights** section, change the coolant temperature's **Rate Weight** from the default `0.1` to `0.3`.
- In the **Output weights** section, change the reactant concentration's **Weight** (last entry in the second row) from the default `1.0` to `0`.

Test these changes using the **T Setpoint** scenario (click the toolbar's **Simulation** button).

Figure 3-12 shows that the CSTR temperature now tracks the setpoint change smoothly, reaching the new value in about 10 time units with no overshoot.

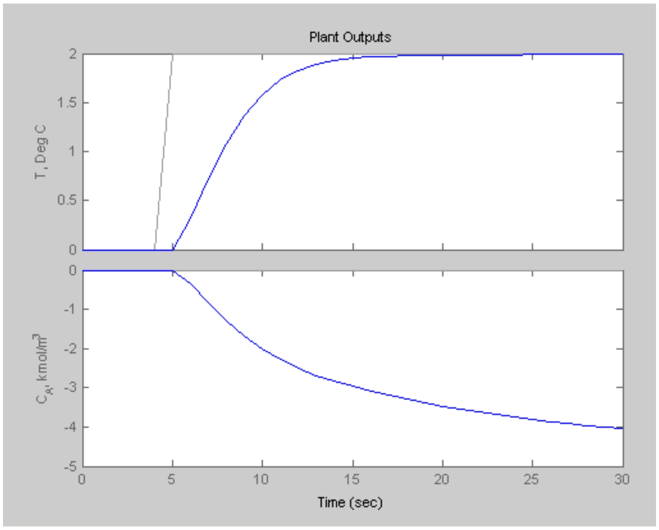


Figure 3-12: Improved Setpoint Tracking for CSTR Temperature

On the other hand, the reactant concentration, C_A, exhibits a larger deviation from its setpoint, which is being held constant at zero (compare to Figure 3-7, where the final deviation is about a factor of 4 smaller).

This behavior reflects an unavoidable trade-off. The controller has only one adjustment at its disposal: the coolant temperature. Therefore, it can't satisfy setpoints on both outputs.

Output Weights

The output weights let you dictate the accuracy with which each output must track its setpoint. Specifically, the controller predicts deviations for each output over the prediction horizon. It multiplies each deviation by the output's weight value, and then computes the weighted sum of squared deviations, $S_y(k)$, as follows

$$S_y(k) = \sum_{i=1}^{P} \sum_{j=1}^{n_y} \left\{ w^y{}_j [r_j(k+i) - y_j(k+i)] \right\}^2$$

where *k* is the current sampling interval, *k+i* is a future sampling interval (within the prediction horizon), *P* is the number of control intervals in the prediction horizon, n_y is the number of plant outputs, w_j^y is the weight for output *j*, and the term $[r_j(k+i) - y_j(k+i)]$ is a predicted deviation for output *j* at interval *k+1*.

The weights must be zero or positive. If a particular weight is large, deviations for that output dominate $S_y(k)$. One of the controller's objectives is to *minimize* $S_y(k)$. Thus, a large weight on a particular output causes the controller to minimize deviations in that output (relative to outputs having smaller weights).

For example, the default values used to produce Figure 3-7 specify equal weights on each output, so the controller is trying to eliminate deviations in both, which is impossible. On the other hand, the design of Figure 3-12 uses a weight of zero on the second output, so it is able to eliminate deviations in the first output.

Note The second output is unmeasured. Its predictions rely on the plant model and the temperature measurements. If the model were reliable, we could hold the predicted concentration at a setpoint, allowing deviations in the reactor temperature instead. In practice, it would be more common to control the temperature as done here, using the predicted reactant concentration as an auxiliary indicator.

You might expect equal output weights to result in equal output deviations at steady state. Figure 3-7 shows that this is not the case. The reason is that the controller is trying to satisfy several additional objectives simultaneously.

Rate Weights

One is to minimize the weighted sum of controller adjustments, calculated according to

$$S_{\Delta u}(k) = \sum_{i=1}^{M} \sum_{j=1}^{n_{mv}} \left\{ w_j^{\Delta u} \Delta u_j(k+i-1) \right\}^2$$

where M is the number of intervals in the control horizon, n_{mv} is the number of manipulated variables, $\Delta u_j(k+i-1)$ is the predicted adjustment in manipulated variable j at future (or current) sampling interval $k+i-1$, and $w_j^{\Delta u}$ is the weight on this adjustment, called the *rate weight* because it penalizes the incremental change rather than the cumulative value. Increasing this weight forces the controller to make smaller, more cautious adjustments.

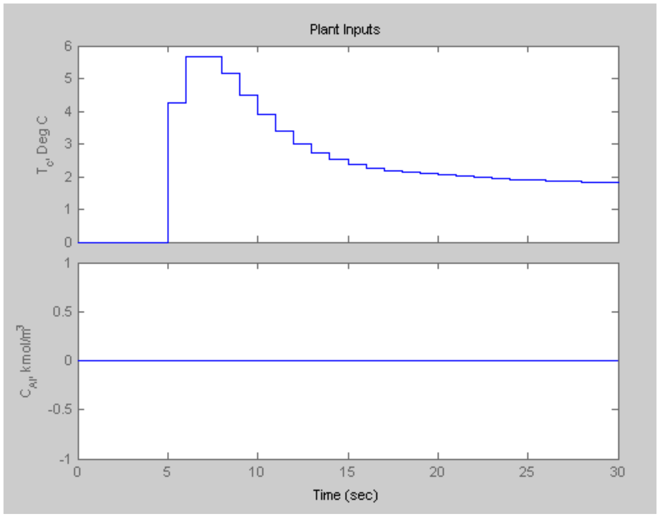


Figure 3-13: Plant Inputs for Modified Rate Weight

Figure 3-13 shows that increasing the rate weight from 0.1 to 0.3 decreases the move sizes significantly, especially the initial move (compare to Figure 3-8).

Setting the rate weight to `0.1` yields a faster approach to the T setpoint with a small overshoot, but the initial T_c move is about six times larger than needed to achieve the new steady state, which would be unacceptable in most applications (not shown; try it).

Note The controller minimizes the sum $S_y(k) + S_{\Delta u}(k)$. Changes in the coolant temperature have unequal effects on the two outputs, so the steady-state output deviations won't be equal, even when both output weights are unity as in Figure 3-7.

Input Weights

The controller also minimizes the weighted sum of manipulated variable deviations from their nominal values, computed according to

$$S_u(k) = \sum_{i=1}^{M} \sum_{j=1}^{n_{mv}} \left\{ w_j^u [u_j(k+i-1) - \bar{u}_j] \right\}^2$$

where w_j^u is the *input weight* and $\bar{u}_j$ is the nominal value for input *j*. In the above simulations, you used the default, $w_j^u = 0$. This is the usual choice.

When a sustained disturbance or setpoint change occurs, the manipulated variable must deviate permanently from its nominal value (as shown in Figure 3-8 and Figure 3-13). Using a nonzero input weight forces the corresponding input back toward its nominal value. Test this by running a simulation in which you set the input weight to 1. The final T_c value is closer to its nominal value, but this causes T to deviate from the new setpoint (not shown).

Note Some applications involve more manipulated variables than plant outputs. In such cases, it is common to define nonzero input weights on certain manipulated variables in order to hold them near their most economical values. The remaining manipulated variables eliminate steady-state error in the plant outputs.

Blocking

The section "Weight Tuning" on page 3-19 used *penalty weights* to shape the controller's response. This section covers the following topics:

- An alternative to penalty weighting, called *blocking*
- Side-by-side controller comparisons

To begin, select **Controllers** in the tree, and click the **New** button, creating a controller initialized to the Model Predictive Control Toolbox™ default settings. Rename this controller **Blocking 1** by editing the appropriate table cell.

Select **Blocking 1** in the tree, select its **Weight Tuning** tab, and set the **Weight** for output `C_A` to `0` (see "Weight Tuning" on page 3-19 to review the reason for this). Leave other weights at their defaults.

Now select the **Model and Horizons** tab, and select its **Blocking** check box. This activates the blocking options. It also deactivates the **Control Horizon** option (the blocking options override it).

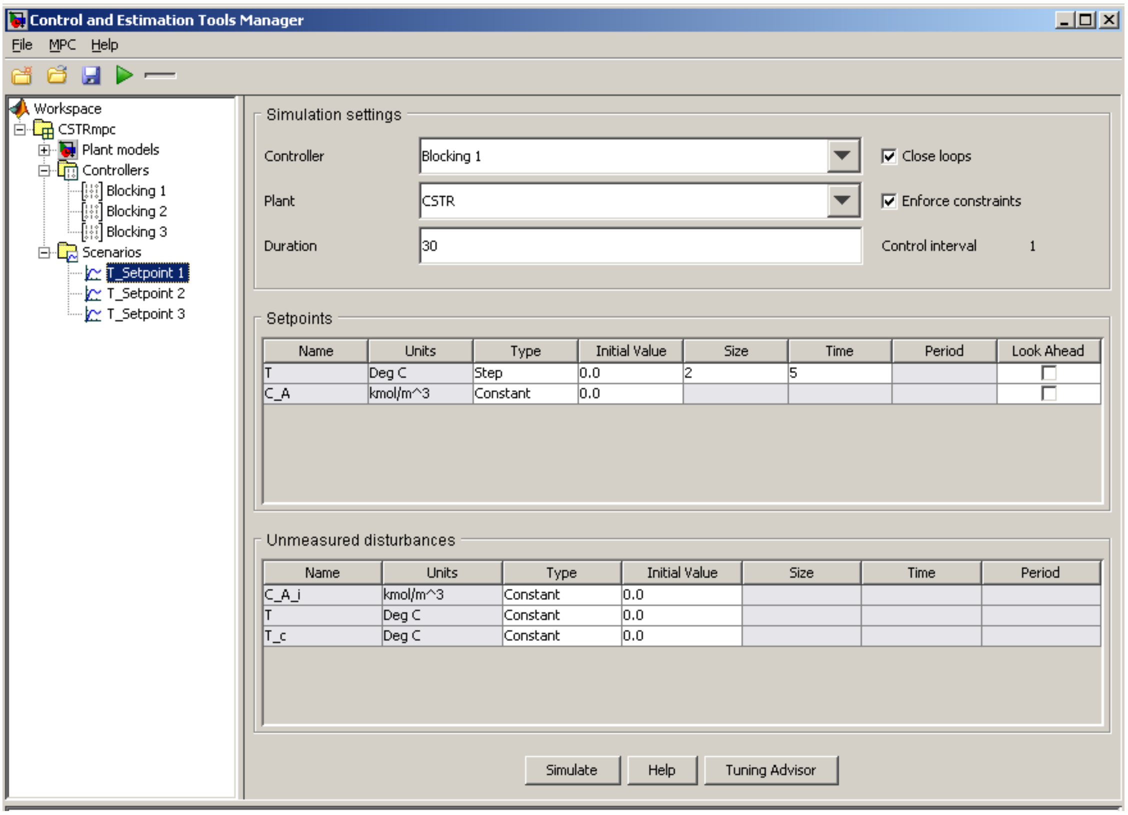
Set **Number of moves computed per step** to `2`. Verify that **Blocking allocation within prediction horizon** is set to `Beginning`, the default.

Select **Controllers** in the tree, and use its **Copy** button to create two controllers based on **Blocking 1**. Rename these **Blocking 2** and **Blocking 3**. Edit their blocking options, setting **Blocking allocation within prediction horizon** to `Uniform` for **Blocking 2**, and to `End` for **Blocking 3**.

Select **Scenarios** in the tree. Rename the **T Setpoint** scenario to **T Setpoint 1**, and set its **Controller** option to **Blocking 1**.

Create two copies of **T Setpoint 1**, naming them **T Setpoint 2** and **T Setpoint 3**. Set their **Controller** options to **Blocking 2** and **Blocking 3**, respectively. Now you should have three scenarios, identical except for the controller being used.

Delete the **MPC1** controller and select **T Setpoint 1** in the tree. Your view should resemble Figure 3-14, with three controllers and three scenarios in the tree.



Simulation settings

Controller	Blocking 1
Plant	CSTR
Duration	30
☑ Close loops	
☑ Enforce constraints	
Control interval	1

Setpoints

Name	Units	Type	Initial Value	Size	Time	Period	Look Ahead
T	Deg C	Step	0.0	2	5		☐
C_A	kmol/m^3	Constant	0.0				☐

Unmeasured disturbances

Name	Units	Type	Initial Value	Size	Time	Period
C_A_i	kmol/m^3	Constant	0.0			
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	0.0			

Figure 3-14: T Setpoint 1 Scenario

If any simulation plot windows are open, close them. This forces subsequent simulations to generate new plots.

Simulate each of the three scenarios. When you run the first, new plot windows open. Leave them open when you run the other two scenarios so all three results appear together, as shown in Figure 3-15 and Figure 3-16.

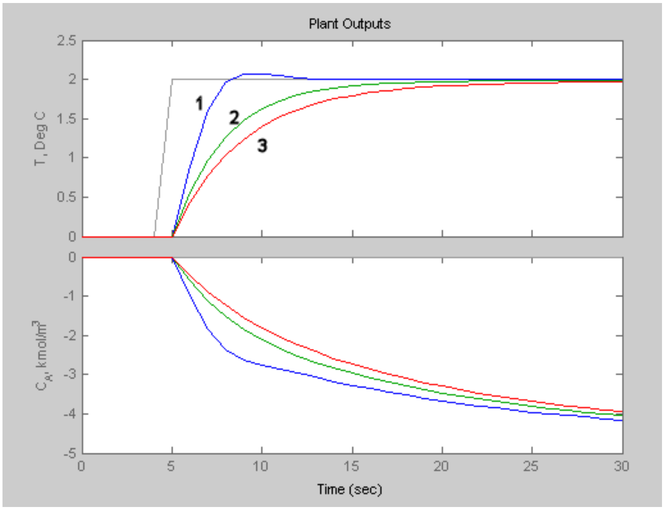


Figure 3-15: Blocking Comparison, Outputs

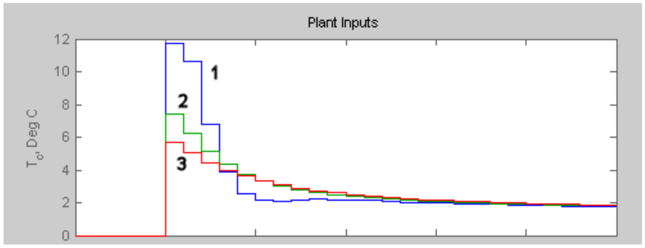


Figure 3-16: Blocking Comparison, Manipulated Variable

The numeric annotations on these figures refer to the three scenarios. Recall that **T Setpoint 1** uses the default blocking, which usually results in faster setpoint tracking but larger manipulated variable moves. The blocking options in **T Setpoint 2** and **T Setpoint 3** reduce the move size but make setpoint tracking more sluggish.

Results for **T Setpoint 3** are very similar to those shown in Figure 3-12 and Figure 3-13, where a penalty *rate weight* reduced the move sizes. If rate weights and blocking achieve the same ends, why does the toolbox provide both features? One difference not evident in this simple problem is that blocking applies to all the manipulated variables in your application, but each rate weight affects one only.

Note To obtain the dashed lines shown for **T Setpoint 2**, activate the plot window and select **Property Editor** from the **View** menu. The Property Editor appears at the bottom of the window. Then select the curve you want to edit. The Property Editor lets you change the line type, thickness, color, and symbol type. Select the axis labels to see additional options.

By default, the toolbox plots each scenario on the same plot. If you recalculate a revised scenario, it replots that result but doesn't change any others.

If you don't want to see a particular scenario, right-click the plot and use the **Responses** menu option to hide it. (You can also close the plot window and recalculate selected responses in a fresh window.)

Defining Manipulated Variable Constraints

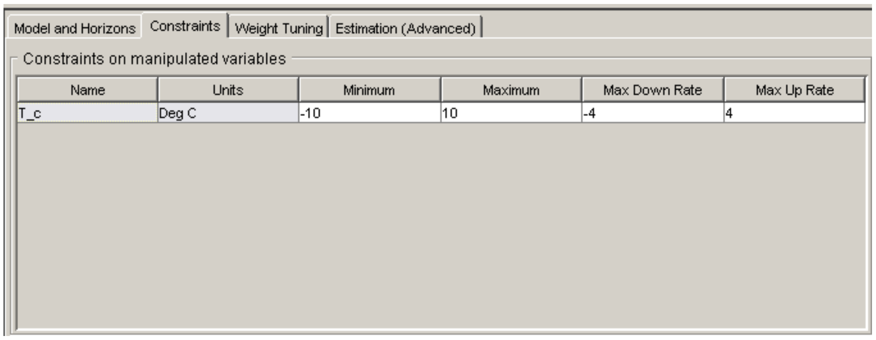
Physical devices have limited ranges and rates of change. For example, the `CSTR` model's coolant might be restricted to a 20 degree range (from -10 to 10) and its maximum rate of change might be ±4 degrees per control interval. If these are true physical restrictions, it's good practice to include them in the controller design. Otherwise the controller might attempt an unrealistic adjustment.

To compare constrained and unconstrained performance for the `CSTR` example, select the **Blocking 1** controller in the tree, and rename it **Unconstrained**.

Select its **Model and Horizons** tab and turn off (clear) its **Blocking** option. Increase **Control horizon** to `3`, and reduce **Control Interval** to `0.25`.

Delete the **Blocking 2** and **Blocking 3** controllers. (Click **Yes** or **OK** to dismiss the resulting warning messages.)

Copy the **Unconstrained** controller. Name this copy **MVconstraints**, and select its **Constraints** tab. Then enter the manipulated variable constraints shown in Figure 3-17.



Name	Units	Minimum	Maximum	Max Down Rate	Max Up Rate
T_c	Deg C	-10	10	-4	4

Figure 3-17: Entering CSTR Manipulated Variable Constraints

If any simulation plot windows are open, close them (to force fresh plots).

Select the **T Setpoint 1** scenario. If necessary, set its **Controller** option to `Unconstrained`. Change **Duration** to `15`, and simulate the scenario.

Select the **T Setpoint 2** scenario, set its **Controller** option to `MVconstraints`, change its **Duration** to `15`, and simulate it. The results appear in Figure 3-18 and Figure 3-19.

The larger control horizon and smaller control interval cause the unconstrained controller to make larger moves (see Figure 3-19, curve 1). The output settles at the new setpoint in about 5 time units rather than the 10 required previously (compare curve 1 in Figure 3-18 to curve 1 in Figure 3-15).

Figure 3-19 (curve 2) shows that the **Max Up Rate** constraint limits the size of the first two moves to 4 degrees. The third move hits the **Maximum** constraint at 10 degrees. The coolant temperature remains saturated at its upper limit for the next 7 control intervals, then slowly moves back down to its final value.

Figure 3-18 (curve 2) shows that the output response is slower, but still settles at the new setpoint smoothly within about 5 time units. This demonstrates the *anti-windup* protection provided automatically by the Model Predictive Control Toolbox controller.

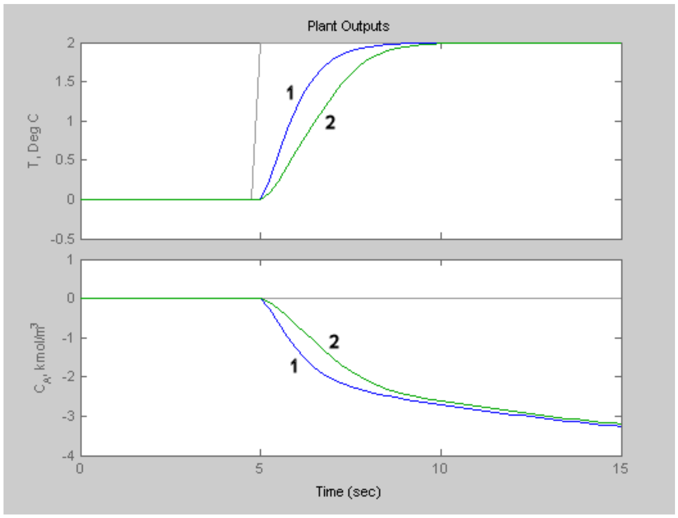


Figure 3-18: CSTR Outputs, Unconstrained (1) and MVconstraints (2)

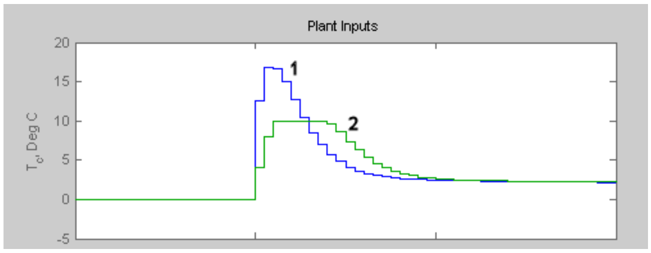


Figure 3-19: CSTR Manipulated Variable, Unconstrained (1) and MVconstraints (2)

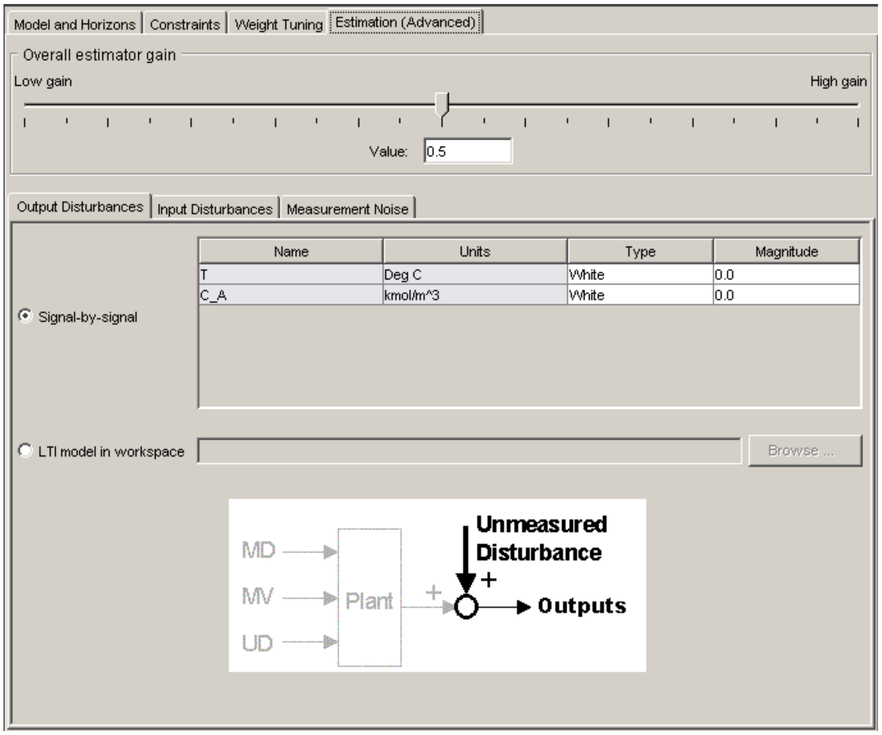
Disturbance Modeling and Estimation

The previous sections tested the controller's response to setpoint changes. In process control, disturbances rejection is often more important. The Model Predictive Control Toolbox product allows you to tailor the controller's disturbance response. The following example assumes that you have just completed the previous section, and the design tool is still open.

Select the first controller in the tree and rename it **InputSteps**. (Its settings should be identical to the **Unconstrained** controller of the previous section.)

Copy this controller. Rename the copy **OutputSteps**. Click its **Estimation** tab. The initial view should be as in Figure 3-20. Note the following:

- Model Predictive Control Toolbox default settings are being used. (The **Use Model Predictive Control Defaults** button restores these settings if you modify them.)
- The **Output Disturbances** tab is selected, and the **Signal-by-signal** option is selected. The graphic shows that the output disturbances add to each output.
- The tabular entries show, however, that these disturbance magnitudes are currently zero.



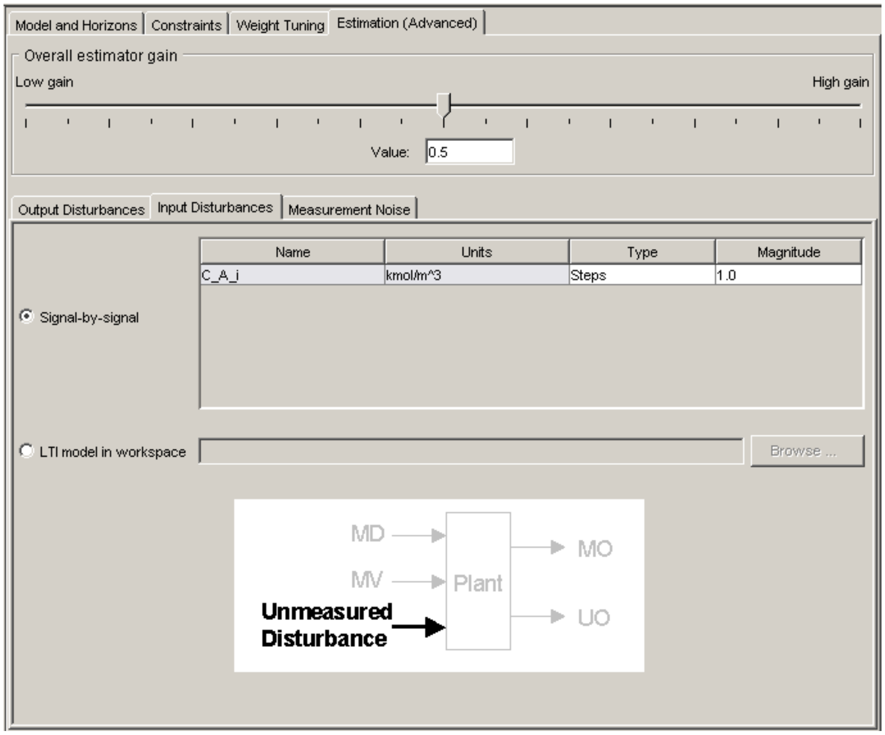
Value: 0.5

● Signal-by-signal

Name	Units	Type	Magnitude
T	Deg C	White	0.0
C_A	kmol/m^3	White	0.0

Figure 3-20: Default Output Disturbance Settings for CSTR

Click the **Input Disturbances** tab. (This would be inactive if the plant model had no unmeasured disturbances.) The view should change to that shown in Figure 3-21.



Value: 0.5

(● Signal-by-signal) ○ LTI model in workspace

Name	Units	Type	Magnitude
C_A_i	kmol/m^3	Steps	1.0

Figure 3-21: Default Input Disturbance Settings for CSTR

In this case the disturbance magnitude is nonzero, and the disturbance type is `Steps`. Thus, the controller assumes that disturbances enter as random steps (integrated white noise) at the plant's unmeasured disturbance input.

Click the **Measurement Noise** tab, verifying that the controller is assuming white noise, magnitude `1` (not shown).

The following summarizes Model Predictive Control Toolbox default disturbance modeling assumptions for this case:

- Additive output disturbances: none

- Unmeasured input disturbances: random steps (integrated white noise)
- Measurement noise: white

In general, if your plant model includes unmeasured disturbance inputs, the toolbox default strategy will assume that they are dominant and sustained, as in the above example. This forces the controller to include an integrating mode, intended to eliminate steady-state error.

If the plant model contains no unmeasured input disturbances, the toolbox assumes sustained (integrated white noise) disturbances at the measured plant outputs.

If there are more measured outputs than unmeasured input disturbances, it assumes sustained disturbances in both locations according to an algorithm described in the product's online documentation.

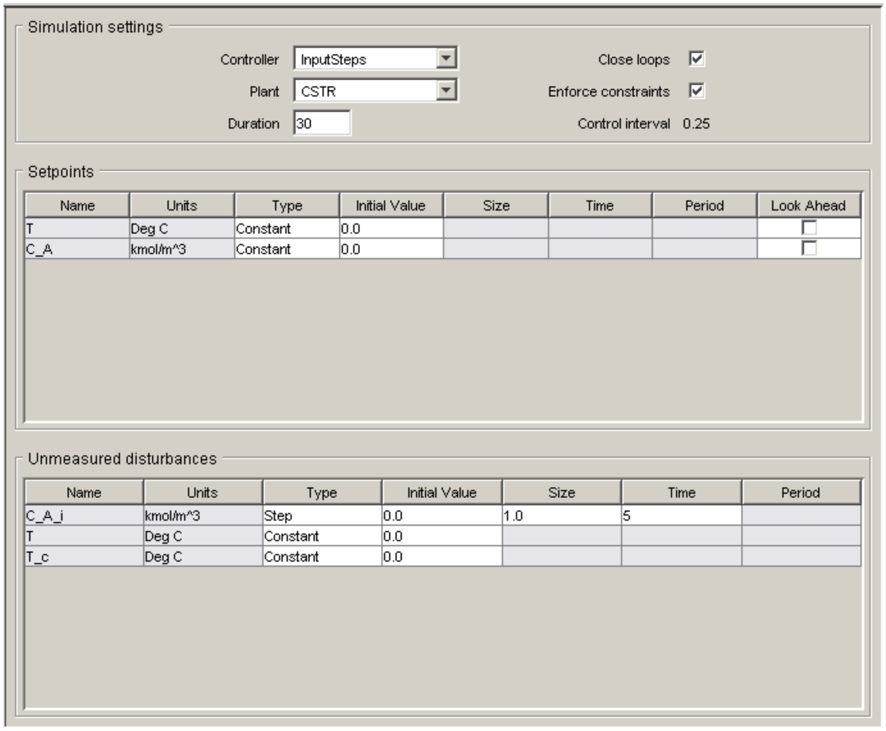
In any case, the design tool displays the assumptions being used.

To modify the estimation strategy in the `OutputSteps` controller, do the following:

- Click the **Input Disturbances** tab. Set the disturbance **Type** to `White`, and its **Magnitude** to `0`.
- Click the **Output Disturbances** tab. For the `T` output, set the disturbance Type to `Steps`, and its magnitude to `1`.

This *reverses* the default assumptions, i.e., the **OutputSteps** controller assumes that sustained disturbances enter at the plant output rather than at the unmeasured disturbance input. The **InputSteps** controller is still using the original (default) assumptions.

Next, select the first scenario in the tree. Rename it **Disturbance 1**, set its **Duration** to `30`, define all setpoints as constant zero values, and define a unit-step disturbance in the unmeasured input, `C_A_i`. If necessary, set its **Controller** option to `InputSteps`. Figure 3-22 shows the final **Disturbance 1** scenario.



Simulation settings

Controller: InputSteps
Plant: CSTR
Duration: 30
Close loops: ✓
Enforce constraints: ✓
Control interval: 0.25

Setpoints

Name	Units	Type	Initial Value	Size	Time	Period	Look Ahead
T	Deg C	Constant	0.0				☐
C_A	kmol/m^3	Constant	0.0				☐

Unmeasured disturbances

Name	Units	Type	Initial Value	Size	Time	Period
C_A_i	kmol/m^3	Step	0.0	1.0	5	
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	0.0			

Figure 3-22: CSTR Disturbance 1 Scenario

Copy **Disturbance 1**. Rename the copy **Disturbance 2**, and set its **Controller** option to `OutputSteps`.

If necessary, close any open simulation plot windows. Simulate both scenarios. Figure 3-23 and Figure 3-24 show the results.

Figure 3-23 shows that default controller (case 1) returns to the setpoint in less than one third the time required by the modified controller (case 2). Its maximum deviation from the setpoint is also 10% smaller. Figure 3-24 shows that in both cases the input moves are smooth and of reasonable magnitude. (It also shows the input disturbance.)

The default controller expects unmeasured disturbances to enter as defined in the scenarios, so it's not surprising that the default controller performs better. The point is that the difference can be large, so it merits design consideration.

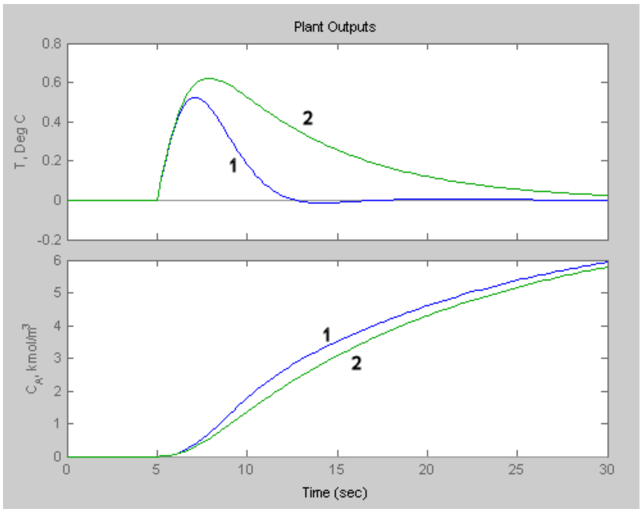


Figure 3-23: CSTR Outputs for Disturbance Scenarios 1 and 2

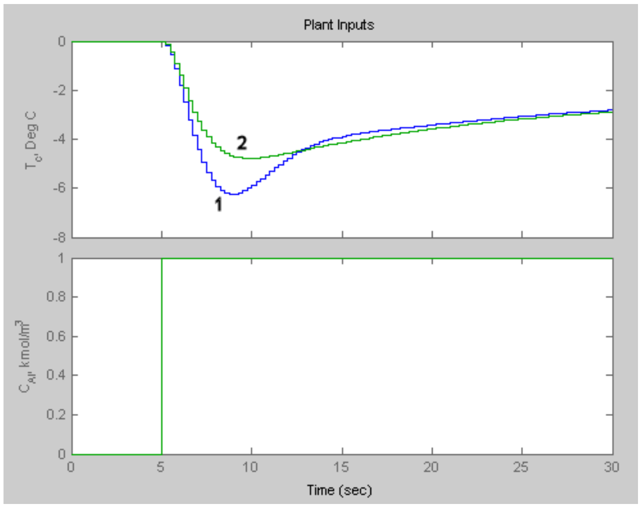


Figure 3-24: CSTR Inputs for Disturbance Scenarios 1 and 2

For comparison, reset the two scenarios so that the only disturbance is a one-degree step increase added to the measured reactor temperature. The modified controller (case 2) is designed for such disturbances, and Figure 3-25 shows that it performs better, but the difference is less dramatic than in the previous scenario. The default controller is likely to be best if the real process has multiple dominant disturbance sources.

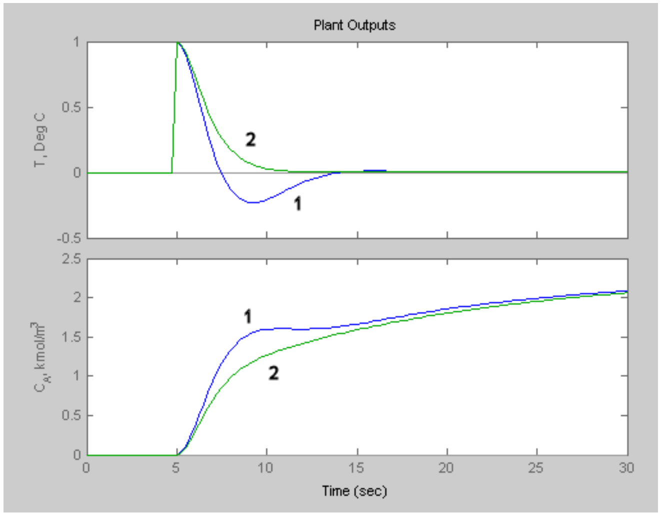


Figure 3-25: CSTR Outputs, Output Disturbance Scenarios 1 and 2

Defining Soft Output Constraints

The discussion in “Weight Tuning” on page 3-19, defined temperature control as the primary goal for the `CSTR` application. The predicted (but unmeasured) reactant concentration, C_A, could vary freely.

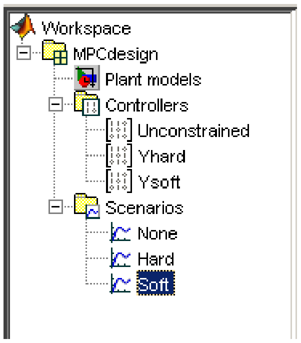
Suppose this were acceptable provided that C_A stayed below a specified maximum (above which unwanted reactions would occur). You can use an *output constraint* to enforce this specification.

Start with a single controller identical to the **InputSteps** controller described in “Disturbance Modeling and Estimation” on page 3-29. Rename it **Unconstrained**.

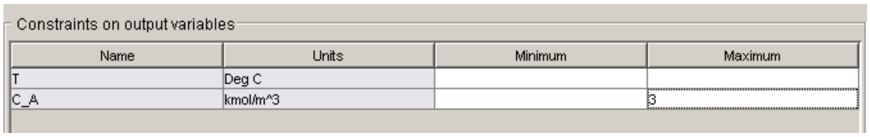
Right-click **Unconstrained** in the tree and select **Copy**. Rename the copy **Yhard**. Make another copy, naming it **Ysoft**.

Similarly, start with a single scenario identical to Figure 3-22, except that its **Controller** setting should be `Unconstrained`. Name this scenario **None**.

Right-click **None** in the tree and select **Copy**. Rename the copy **Hard**, and change its **Controller** setting to `Yhard`. Make another copy, naming it **Soft** and changing its **Controller** setting to `Ysoft`. Your tree should be as shown below.

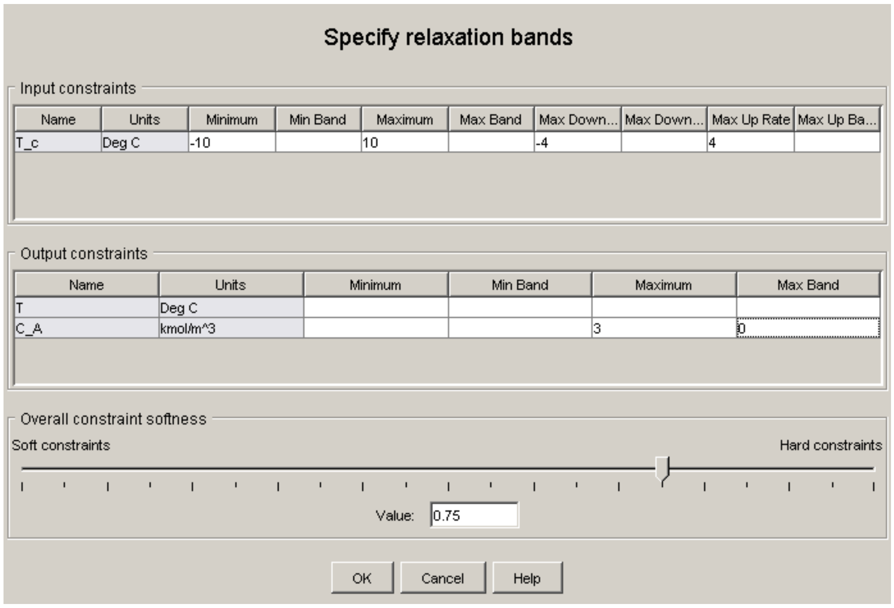


Select your **Yhard** controller. On the **Constraints** tab, set the maximum for C_A to 3 as shown below.


Constraints on output variables

Name	Units	Minimum	Maximum
T	Deg C		
C_A	kmol/m^3		3

Click the **Constraint Softening** button to open the dialog box in Figure 3-26. The **Input constraints** section shows the bounds on the inputs and their rates, and *relaxation bands*, which let you *soften* these constraints. By default, input constraints are *hard*, meaning that the controller tries to prevent any violation.



Input constraints

Name	Units	Minimum	Min Band	Maximum	Max Band	Max Down...	Max Down...	Max Up Rate	Max Up Ba...
T_c	Deg C	-10		10		-4		4	

Output constraints

Name	Units	Minimum	Min Band	Maximum	Max Band
T	Deg C				
C_A	kmol/m^3			3	0

Overall constraint softness

Soft constraints — Hard constraints

Value: 0.75

Figure 3-26: Constraint Softening Dialog Box

The **Output constraints** section lists the output limits and their relaxation bands. By default, the output constraints are *soft*. Make the C_A upper limit hard by entering a zero as its relaxation band (as in Figure 3-26).

Select the **Ysoft** controller. Define a soft upper bound on C_A by using the same settings shown in Figure 3-26, but with a relaxation band of 100 instead of 0.

Simulate the three scenarios in the order they appear in the tree, i.e., **None**, **Hard**, **Soft**. The resulting output responses appear in Figure 3-27.

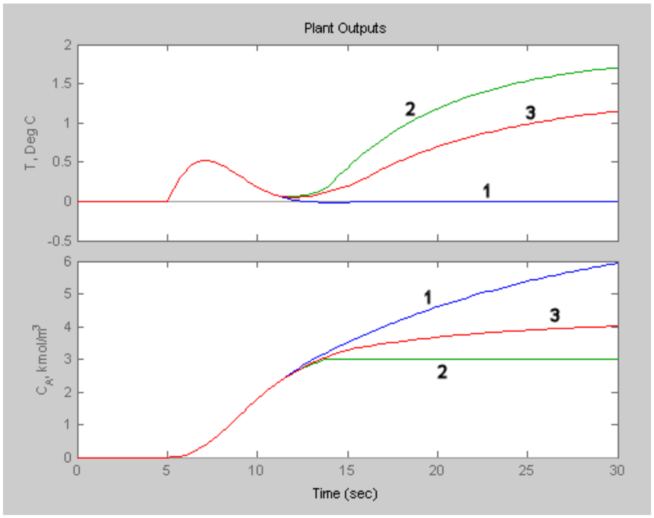


Figure 3-27: Constraint Softening Scenarios: 1 = None, 2 = Hard, 3 = Soft

Curve 1 is without output constraints, which is identical to curve 1 in Figure 3-23. This controller allows the C_A output to vary freely, but the controlled temperature returns to its setpoint within 10 time units after the disturbance happens.

Curve 2 shows the behavior with a hard upper limit at $C_A = 3$. Once C_A reaches this bound, the controller must use its one manipulated variable (T_c) to satisfy the constraint, so it's no longer able to control the temperature.

Curve 3 shows the result for a soft constraint. The controller reaches a compromise between the competing objectives: temperature control and constraint satisfaction. As you'd expect, performance lies between the curve 1 and curve 2 extremes.

The numerical value of the relaxation band represents a *relative tolerance* for constraint violations, not a strict limit (if it were the latter, it would be a hard constraint). If you were to increase its relaxation band (currently set at 100), performance would move toward Case 1, and vice versa.

If you have multiple constraints, you can harden or soften them simultaneously using the slider at the bottom of the controller's constraint softening dialog box (see Figure 3-26).

In general, you'll have to experiment to determine the settings that provide appropriate trade-offs for your application. In particular, the relaxation band settings interact with those on the controller's **Weight Tuning** tab (see "Weight Tuning" on page 3-19).

Another important factor is the expected numerical range for each variable. For example, if a particular variable stays within ± 0.1 of its nominal value, it should have a small relaxation band relative to another variable having a range of ± 100.

For details on the Model Predictive Control Toolbox™ constraint softening formulation, see "Optimization Problem" on page 2-5.

Robustness Testing

It's good practice to test your controller's sensitivity to prediction errors. Classical phase and gain margins are one way to quantify robustness for a SISO application. Robust Control Toolbox™ software provides sophisticated approaches for MIMO systems. It can also be helpful to run simulations. The following example illustrates the simulation approach.

Plant Model Perturbation

Use the following code to create a perturbed version of the CSTR model:

```
CSTRp = CSTR;
CSTRp.a=[-0.0303   -0.0113
         -0.0569   -0.1836];
CSTRp.b=[-0.0857    0.0191
          0.1393    0.4241];
```

This creates a copy of CSTR called CSTRp, then replaces the state space *A* and *B* matrices with perturbed versions (compare to the originals defined in "State-Space Format" on page 2-5). Use the following command to compare the two step responses:

```
step(CSTR, CSTRp)
```

Observe the difference in the responses (not shown).

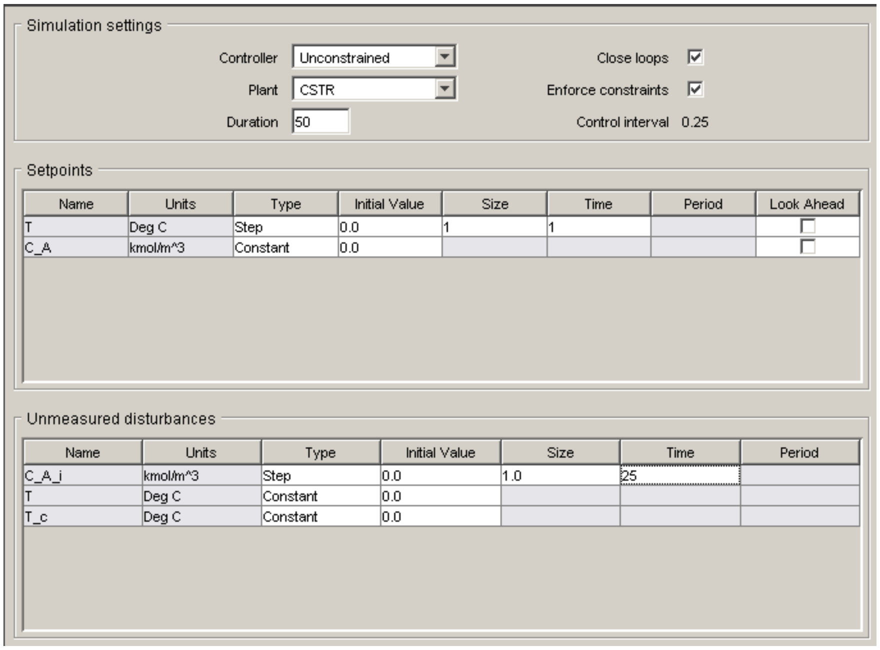
Select **Plant models** in the tree. Click the **Import** button and import the CSTRp model.

Simulation Tests

Delete all controllers except the first one in the tree. If necessary, make its settings identical to **Unconstrained** (see "Defining Manipulated Variable Constraints" on page 3-27).

Delete all scenarios except the first, naming that **Accurate Model**. Define its properties as shown in Figure 3-28. The scenario begins with a step change in the temperature setpoint, followed 25 time units later by a step disturbance in the reactant entering the CSTR.

Copy **Accurate Model**. Rename the copy **Perturbed Model**, and set its **Plant** option to CSTRp. Thus, both scenarios use the same controller, which is based on the CSTR model, but the **Perturbed Model** scenario uses a different model to represent the “real” plant. This tests the controller’s robustness to a change in plant parameters.



Simulation settings

Controller	Unconstrained	Close loops	✓
Plant	CSTR	Enforce constraints	✓
Duration	50	Control interval	0.25

Setpoints

Name	Units	Type	Initial Value	Size	Time	Period	Look Ahead
T	Deg C	Step	0.0	1	1		☐
C_A	kmol/m^3	Constant	0.0				☐

Unmeasured disturbances

Name	Units	Type	Initial Value	Size	Time	Period
C_A_i	kmol/m^3	Step	0.0	1.0	25	
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	0.0			

Figure 3-28: Robustness Test, Accurate Plant Model Scenario

Simulate the two scenarios. Figure 3-29 shows the output responses. As expected, setpoint tracking degrades when the model is inaccurate, but performance is still acceptable.

The disturbance rejection appears to *improve* with the perturbed model. This is a consequence of the perturbations used. The gain for the T/C_{Ai} output/input pair is about 15% smaller in the CSTRp model, which has two beneficial effects: the actual impact of the disturbance is reduced, and the controller is aggressive because it expects a larger impact.

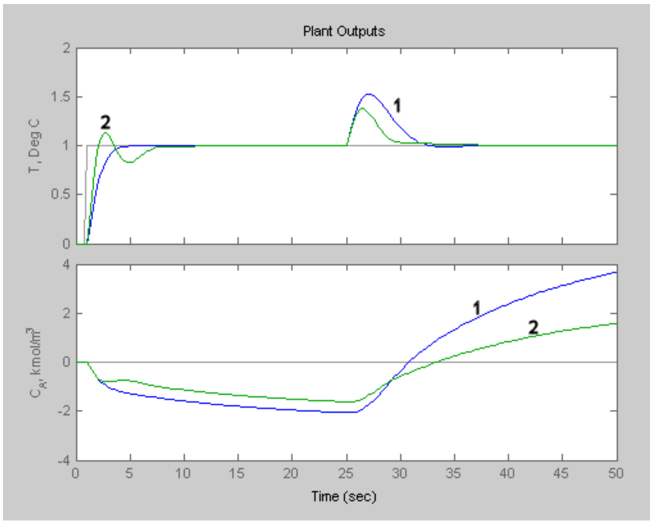


Figure 3-29: Robustness Test, Accurate Model (1) and Perturbed Model (2)

Note MIMO applications are usually more sensitive to model error than SISO applications, so robustness testing is especially recommended for MIMO cases.

Plant Models with Delays

Unlike many controller design approaches, the Model Predictive Control Toolbox can handle models that include delays. A typical example is the distillation column model, `DC`, introduced in "Multiinput Multioutput (MIMO) Plants" on page 2-11. The model includes a delay in each input/output channel.

The presence of delays will influence controller performance, and your controller specifications should account for them. The following example provides some guidelines and covers the following topics:

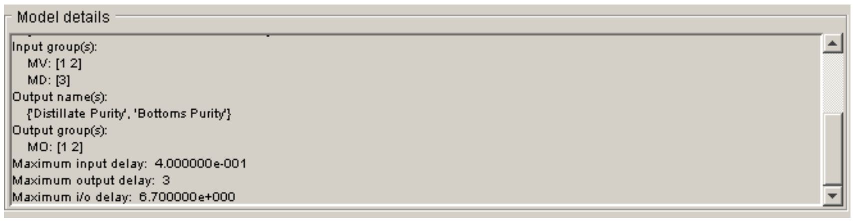
- "Importing the Plant Model" on page 3-43
- "Specifying Controller Horizons" on page 3-43

Importing the Plant Model

To learn how to create the `DC` model, see "Multiinput Multioutput (MIMO) Plants" on page 2-11. You must import the `DC` model into the MATLAB® workspace.

Start the Model Predictive Control Toolbox design tool (type `mpctool` at the MATLAB command line). Import the `DC` model (see "Loading a Plant Model" on page 3-3).

Select **Plant models** in the tree. The `DC` model should be the only one listed. Scroll the **Model details** view to show the last few lines, which should appear as in Figure 3-30.



```
Input group(s):
   MV: [1 2]
   MD: [3]
Output name(s):
   {'Distillate Purity', 'Bottoms Purity'}
Output group(s):
   MO: [1 2]
Maximum input delay:  4.000000e-001
Maximum output delay:  3
Maximum i/o delay:  6.700000e+000
```

Figure 3-30: DC Model Details, Maximum Delay Values

Specifying Controller Horizons

The model's maximum I/O delay is 6.7 minutes. It is good practice to specify the prediction and control horizons such that

$$P - M \gg t_{d,\,max}/\Delta t$$

where P is the prediction horizon, M is the control horizon, $t_{d,max}$ is the maximum delay, and Δt is the control interval.

Select **MPC1** (the default controller name) in the tree. Click the **Model and Horizons** tab, and set **Control interval** to 1, a reasonable choice if the closed-loop response time is to be of order 5-10 minutes.

Given the amount of plant delay and the specified control interval, the default horizons, $P = 10$, $M = 2$, would be a poor choice. Instead, set **Prediction horizon** to 30, and **Control horizon** to 5.

Select **Scenario1** in the tree. Set **Duration** to 50. Define a constant setpoint of 1 for the first output (the distillate purity). Define a step increase of 1 in the second output's setpoint, occurring at $t = 25$. All other signals should be zero. Simulate the scenario. Figure 3-31 and Figure 3-32 show the results.

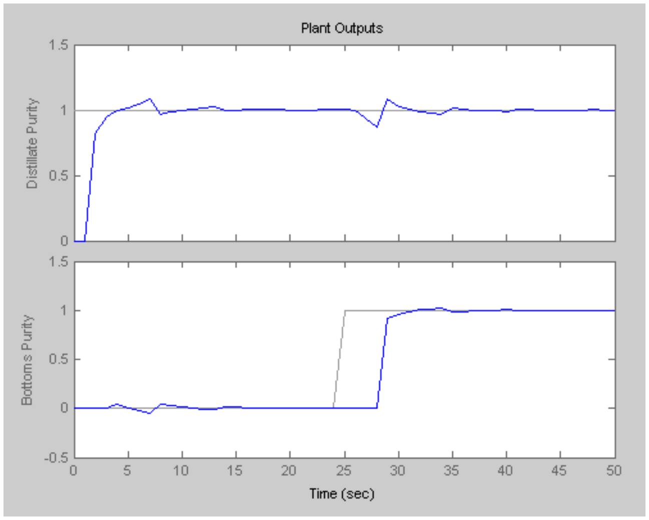


Figure 3-31: DC Setpoint Response Scenario, Outputs

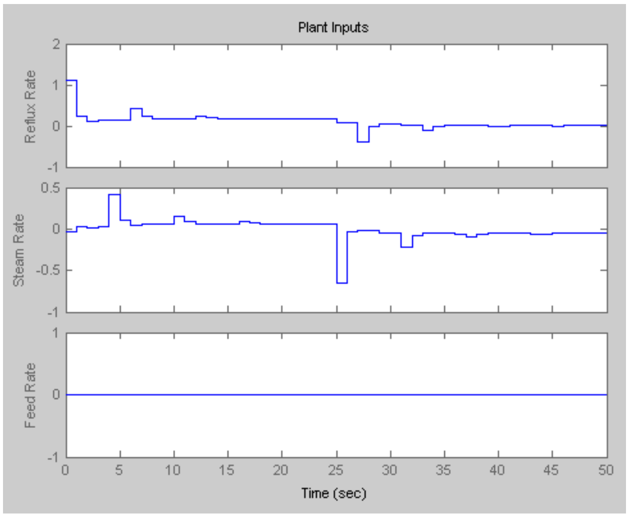


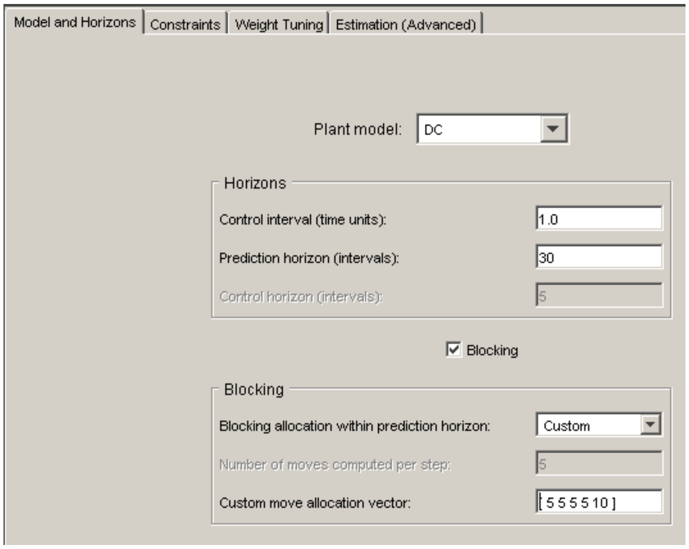
Figure 3-32: DC Setpoint Response Scenario, Inputs

As seen in Figure 3-31, the first output cannot respond for a minimum of one minute, the delay in the y_1/u_1 transfer function. After that, it reaches the setpoint in two minutes and settles quickly. Similarly, y_2 cannot respond for a minimum of three minutes, the delay in the y_2/u_2 transfer function, but settles rapidly thereafter. Changing one setpoint disturbs the other output, but the magnitude of this *interaction* is less than 10%.

Figure 3-32 shows that the initial input moves are more than five times the final change. Also, there are periodic pulses in the control action as the controller attempts to counteract the delayed effects of each input on the two outputs.

You can moderate these effects using the weights (see “Weight Tuning” on page 3-19). Instead, define a custom blocking strategy as illustrated in Figure 3-33. This uses five moves as before, but allocates them more uniformly over the prediction horizon.

Figure 3-34 and Figure 3-35 show the corresponding simulation results. The initial input moves are much smaller, and the moves are less oscillatory overall. The trade-off is a slower output response with about 20% interaction.



Plant model: DC

Control interval (time units): 1.0

Prediction horizon (intervals): 30

Control horizon (intervals): 5

[✓] Blocking

Blocking allocation within prediction horizon: Custom

Number of moves computed per step: 5

Custom move allocation vector: [5 5 5 5 10]

Figure 3-33: DC Model, Custom Blocking Strategy

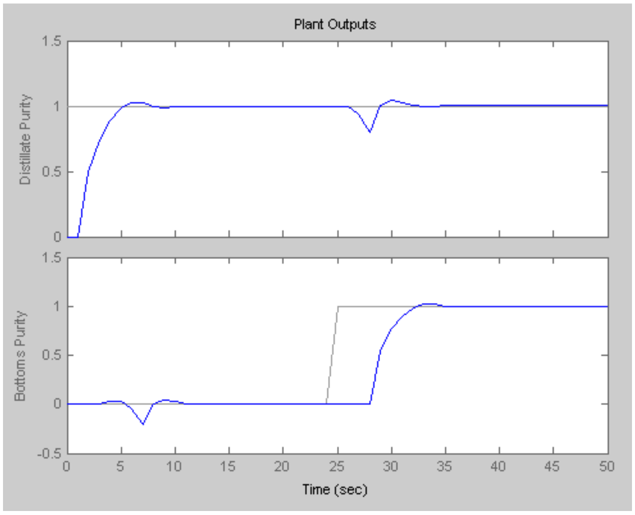


Figure 3-34: Output Responses for Setpoint Scenario with Blocking

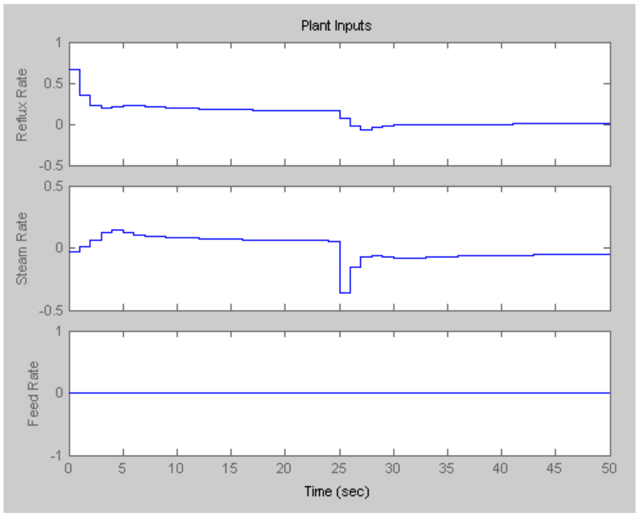


Figure 3-35: Input Moves for Setpoint Scenario with Blocking

Nonsquare Plants

A *nonsquare* plant has an unequal number of manipulated variables and output variables. This is common in practice, and the Model Predictive Control Toolbox™ software supports an excess of manipulated variables or outputs. In such cases you will usually need to modify default toolbox settings.

This section covers the following topics:

- "More Outputs Than Manipulated Variables" on page 3-48
- "More Manipulated Variables Than Outputs" on page 3-49

More Outputs Than Manipulated Variables

When there are excess outputs, you can't hold each at a setpoint. You have the following options:

1. Enforce setpoints on all outputs, in which case all will deviate from their setpoints to some extent.

2. Specify that certain outputs need not be held at setpoints by setting their weights to zero on the controller's **Weight Tuning** tab.

The initial test of the `CSTR` controller used option 1 (the default), which caused both outputs to deviate from their setpoints (see Figure 3-7). You can adjust the offset in each output by changing the output weights. Increasing an output weight decreases the offset in that output (at the expense of increased offset in other outputs).

The modified `CSTR` controller used option 2 (see the discussion in "Weight Tuning" on page 3-19). In general, if the application has N_e more outputs than manipulated variables, setting N_e output weights to zero should allow the remaining outputs to be held at setpoints (unless the manipulated variables are constrained). This was the case for the modified `CSTR` controller (see Figure 3-12).

Outputs that have been "sacrificed" by setting their weights to zero can still be useful. If measured, they can help the controller to estimate the plant's state, thereby improving its predictions. They can also be used as indicators, or as variables to be held within an operating region defined by output constraints.

More Manipulated Variables Than Outputs

In this situation, default Model Predictive Control Toolbox settings should provide offset-free output-setpoint tracking, but the manipulated variables are likely to drift.

One way to avoid this is to use *manipulated variable setpoints*. If there are N_e excess manipulated variables and you hold N_e of them at target values, the rest should not drift. Rather, they will attain the values needed to eliminate output offset.

To define a manipulated variable setpoint:

1 Enter the setpoint value in the **Nominal** field in the signal properties view – see Figure 3-3.

2 Assign a nonzero *input weight* using the **Weight** entry on the controller's **Weight Tuning** tab – see Figure 3-11.

In step 2, the magnitude of the input weight determines the extent to which the manipulated variable can deviate from its target during a transient. See "Input Weights" on page 3-23 for more discussion and mathematical details.

You might want to allow such deviations temporarily in order to provide better output setpoint tracking. In that case, use a relatively small input weight. If you want the manipulated variable to stay near its target value at all times, increase its input weight.

Another way to avoid drift is to constrain one or more manipulated variables to a narrow operating region. You can even hold an MV constant by setting its lower and upper bounds to the same value (in which case its nominal value should also be set to this value), or by setting both of its rate constraints to zero. To define constraints, use the controller's **Constraints** tab (see "Defining Manipulated Variable Constraints" on page 3-27).

Nonlinear Plants

You can control a Simulink® plant modeled using a Model Predictive Control Toolbox™ Simulink block. This section illustrates the procedure using the nonlinear `CSTR` model introduced earlier (see "Using Simulink® to Develop LTI Models" on page 2-19). It covers the following topics:

- "MPC Controller Block" on page 3-50
- "Initiating the Controller Design" on page 3-51
- "Validating the Linearized Model" on page 3-53
- "Modifying the Linearized Model" on page 3-55
- "Linear Simulation Tests" on page 3-56
- "Nonlinear Simulation Tests" on page 3-58
- "Modifying the Controller Using the Design Tool" on page 3-60
- "Exiting the Design Tool" on page 3-60

MPC Controller Block

Figure 3-36 shows `CSTR` with an added MPC Controller block. (The model is called `CSTR_MPC.mdl` in the Model Predictive Control Toolbox `demos` folder.) The intent is to hold the residual concentration at a setpoint by adjusting the coolant temperature. The feed concentration and temperature inputs are disturbances, and the reactor temperature is an auxiliary, uncontrolled output. The concentration has a setpoint (reference), currently fixed at 8.57.

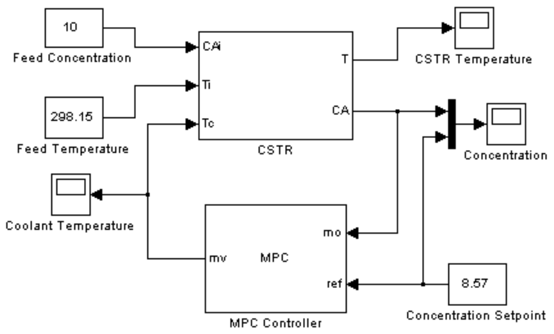
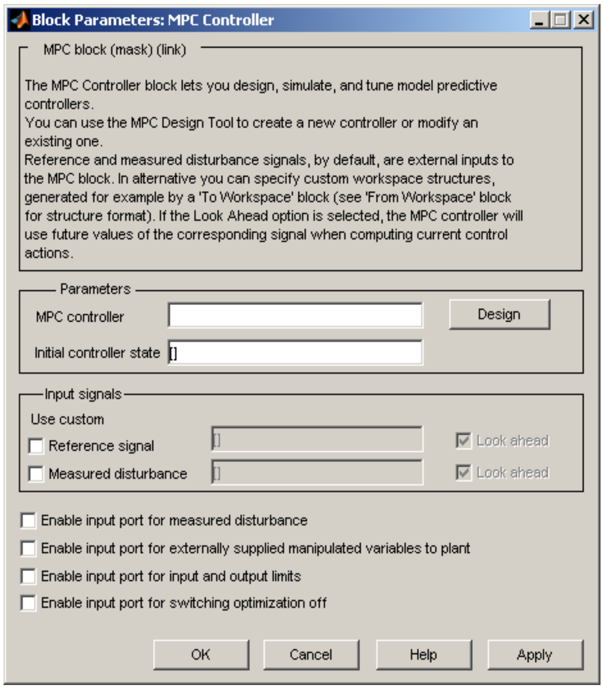


Figure 3-36: Simulink® Model for CSTR Control

Note If you have Model Predictive Control Toolbox software and Simulink installed, the MPC Controller block appears in your Simulink block library. If necessary, see the Simulink documentation for details on how to use library blocks to construct a model.

Initiating the Controller Design

Once you've installed an MPC Controller block in your Simulink model, as in Figure 3-36, you need to configure it. Click it to open its *block mask*, and verify that it is as shown below.


Block Parameters: MPC Controller

MPC block (mask) (link)

The MPC Controller block lets you design, simulate, and tune model predictive controllers.
You can use the MPC Design Tool to create a new controller or modify an existing one.
Reference and measured disturbance signals, by default, are external inputs to the MPC block. In alternative you can specify custom workspace structures, generated for example by a 'To Workspace' block (see 'From Workspace' block for structure format). If the Look Ahead option is selected, the MPC controller will use future values of the corresponding signal when computing current control actions.

Parameters

MPC controller:
Initial controller state: []

Input signals

Use custom

[ ] Reference signal [] [✓] Look ahead
[ ] Measured disturbance [] [✓] Look ahead

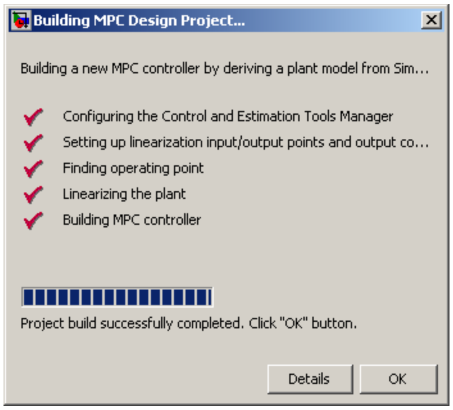
[ ] Enable input port for measured disturbance
[ ] Enable input port for externally supplied manipulated variables to plant
[ ] Enable input port for input and output limits
[ ] Enable input port for switching optimization off

Figure 3-37: MPC Controller Block Mask

If you had used the design tool to create a controller and had saved it in your workspace, you could type its name in the **MPC controller** box and you'd be ready to test it.

Instead, create the controller using the Model Predictive Control Toolbox design tool. To do so, click the **Design** button. A dialog box (not shown) asks you to enter the number of *manipulated variables*, which are the plant inputs the controller can adjust to achieve its objectives. The coolant temperature is the only manipulated variable. Verify that the dialog box entry is 1 and click **OK**.

The following progress window marks the completion of each controller initialization step.


Building MPC Design Project...

Building a new MPC controller by deriving a plant model from Sim...

✓ Configuring the Control and Estimation Tools Manager
✓ Setting up linearization input/output points and output co...
✓ Finding operating point
✓ Linearizing the plant
✓ Building MPC controller

Project build successfully completed. Click "OK" button.

They are as follows:

1. Open the Control and Estimation Tools Manager, and install a *project* having the same name as the Simulink model (`CSTR_MPC` in this case).

2. Install a *linearization task* within the project and determine the plant's I/O points, i.e., the signals connected to the MPC Controller block's `mv` (manipulated variable) and `mo` (measured output) ports.

3. Determine a default *operating point*. This requires assumptions that might be incorrect, as demonstrated in the next section. In particular, the default outputs are zero.

4. Calculate a *linearized plant model* at this operating point using the linearization tool in Simulink® Control Design™ (see "Linearization Using

Simulink® Control Design™" on page 2-19). The controller is open loop during this step and involves only the blocks between the plant inputs and outputs (as determined in step 2).

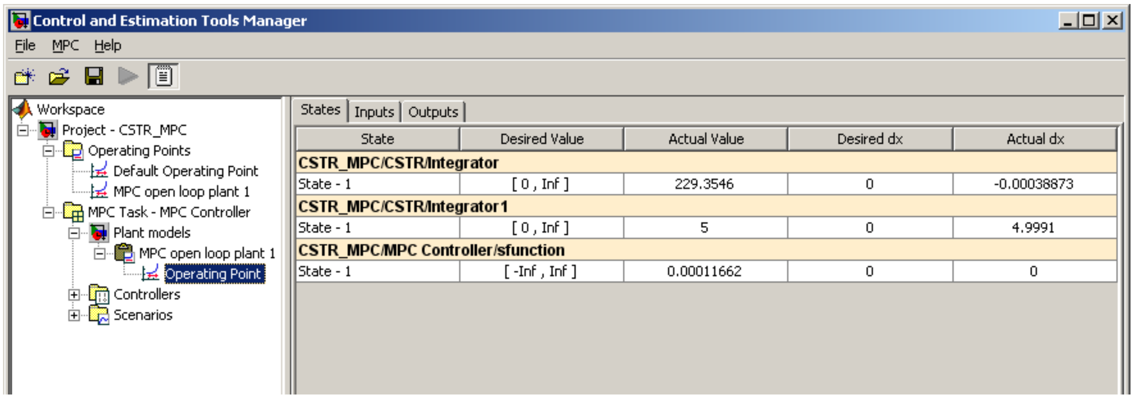
5 Install a Model Predictive Control Toolbox design task in the project. Use the linearized model from step 4 to define a default controller (named **MPC1**), and enter its name as the block mask's **MPC controller** parameter.

When the last step has been completed, click **OK** to close the progress window.

Validating the Linearized Model

Automatic linearization requires assumptions that might be incorrect. You should always validate a linearized model before using it in a controller.

As shown below, the controller task is the **MPC Controller** node, the name of the corresponding Simulink block.



State	Desired Value	Actual Value	Desired dx	Actual dx
CSTR_MPC/CSTR/Integrator				
State - 1	[0 , Inf]	229.3546	0	-0.00038873
CSTR_MPC/CSTR/Integrator1				
State - 1	[0 , Inf]	5	0	4.9991
CSTR_MPC/MPC Controller/sfunction				
State - 1	[-Inf , Inf]	0.00011662	0	0

Note In general, a Simulink model can contain multiple controllers, in which case the project would include multiple controller design tasks, each with a unique name.

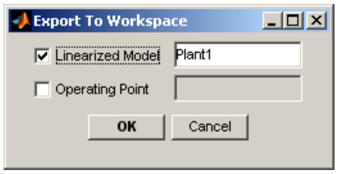
Start by examining the default operating point. Expand the tree until it appears as above, and then select **Operating Point**. The **States** tab should be active as shown (if not, click it).

By default, the Simulink Control Design linearization tool tries to find a steady-state point, i.e., it sets the **Desired dx** column to zero. As shown above, the **Actual dx** column contains one value that is far from zero (4.9991), i.e., it has failed to achieve this goal.

Note Don't be concerned if you see numerical values that differ slightly from those shown.

Next, click the **Outputs** tab. By default, the linearization tool sets the desired plant output to zero. In the CSTR, this would require 100% conversion of the reactant, which is impossible, and the **Outputs** tab confirms that the desired values were not achieved.

Finally, export the linearized model to your workspace. Right-click the **MPC open loop plant 1** node and choose **Export** to open the following dialog box.



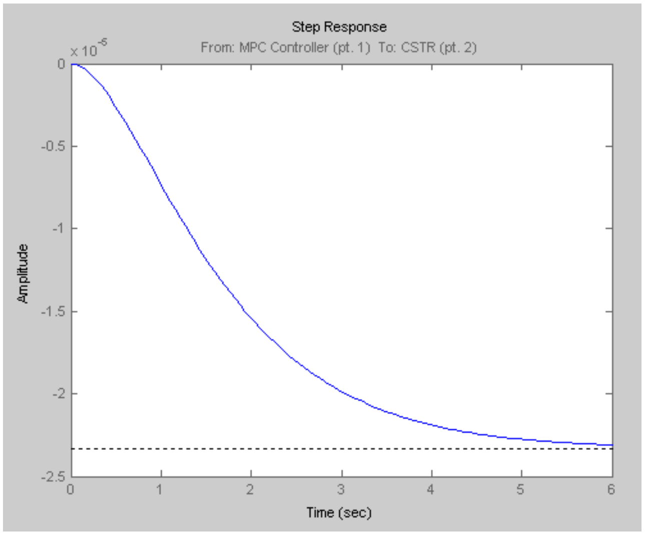
☑ Linearized Model: Plant1
☐ Operating Point

Rename **Linearized Model** to `Plant1` as shown, clear the **Operating Point** check box (because there's no reason to export it), and click **OK**. This exports the linearized model as an LTI object named `Plant1`.

At the MATLAB® command line, type:

```
step(Plant1)
```

You should obtain the following plot.



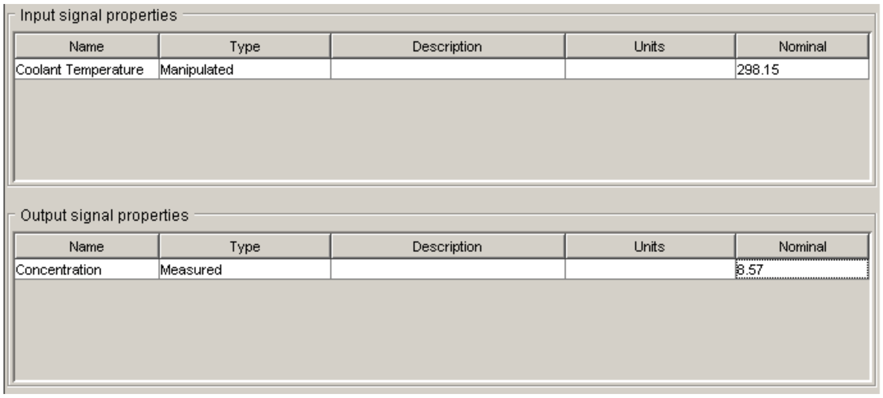
Thus, the linearized model predicts that a unit step *increase* in the coolant temperature will *decrease* the residual concentration, which is qualitatively correct. (Increasing the coolant temperature increases the reactor temperature. This increases the reaction rate, and the residual concentration decreases.)

The predicted magnitude is very small, however: of order 10^{-6}. It should be of order 10^{-2} (you can verify this by removing the controller block from the diagram and running an open-loop step-response simulation). The incorrect assumptions used to generate the default operating point are the cause. If this incorrect model were used in the controller, the coolant-temperature adjustments would be far too large, probably resulting in unstable behavior.

Modifying the Linearized Model

Therefore, the next step is to define a reasonable operating point and calculate a corresponding linearized model.

In the navigation tree, click the **MPC Task - MPC Controller** node. Notice that the nominal input and output signal values are zero by default. As discussed in "Linearization Using Simulink® Control Design™" on page 2-19, the plant is at steady state at a coolant temperature of 298.15 K and a residual concentration of 8.57 kmol/m^3. Thus, in the **Input signal properties** table, set the coolant temperature's nominal value to `298.15`, as shown below, and in the **Output signal properties** table, set the concentration's nominal value to `8.57`. Also assign more descriptive signal names, as shown.



Input signal properties

Name	Type	Description	Units	Nominal
Coolant Temperature	Manipulated			298.15

Output signal properties

Name	Type	Description	Units	Nominal
Concentration	Measured			8.57

Click the **Import Plant** button to open the Plant Model Importer dialog box. Then do the following:

1 Click the **Linearized Plant from Simulink** tab.

2 Change the **Linearization model name** to `Plant2`.

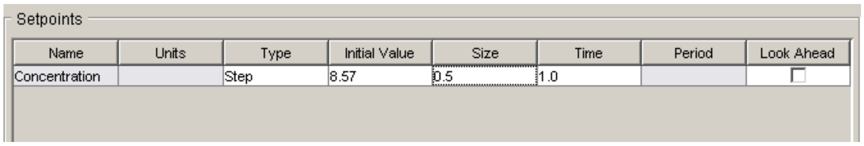
3 Select **Create a new operating condition from MPC I/O values**, and then click **OK**. A new model node named **Plant2** and its operating point will appear within your design task. Verify that its actual dx values are now all close to zero and the outputs are essentially equal to their desired values.

4 Expand the **Controllers** node to expose the **MPC1** node and select it. On its **Model and Horizons** pane, set **Plant model** to `Plant2`. This replaces the default (invalid) model with the modified one.

Leave the other controller settings at their default values.

Linear Simulation Tests

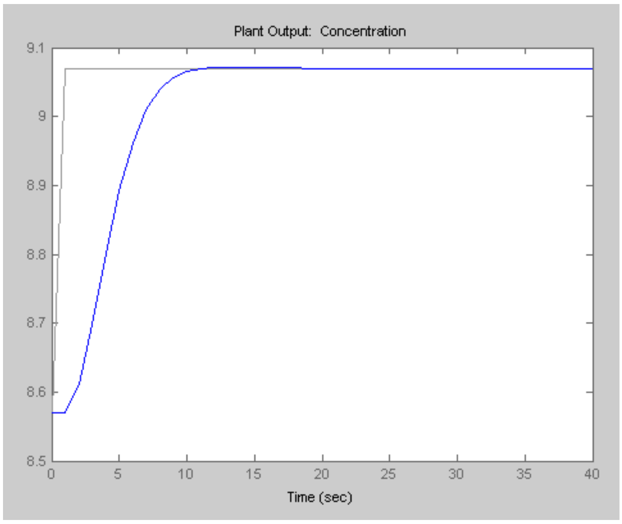
It's good practice to test a controller in linear simulations before trying it on the nonlinear plant. Expand the **Scenarios** node and select **Scenario1**. Set **Plant** to `Plant2`, and set **Duration** to `40`. Finally, define a step increase of `0.5` units in the concentration setpoint starting at time t = 1, as shown below.



Setpoints

Name	Units	Type	Initial Value	Size	Time	Period	Look Ahead
Concentration		Step	8.57	0.5	1.0		☐

Click the scenario's **Simulate** button. This linear test predicts a smooth, rapid approach to the new setpoint with minimal overshoot, as shown below.



The corresponding coolant temperature adjustments are reasonable (not shown). If anything, you might want to make the controller more aggressive by adjusting its tuning weights (see "Weight Tuning" on page 3-19).

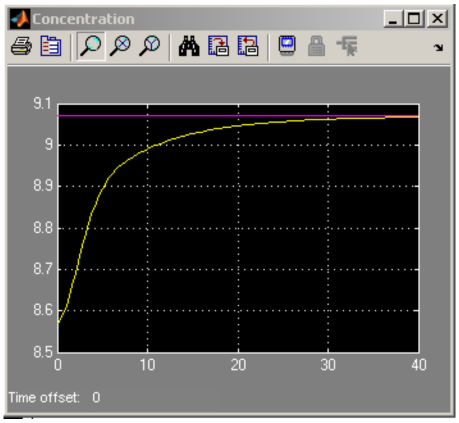
Note This plant's steady-state gain is of order 0.01. Therefore, the default tuning weights lead to a relatively sluggish response. In general, you must adjust the tuning weights to compensate for the plant's natural input/output response magnitudes.

You can also run tests to verify that the controller responds rapidly to either of the two unmeasured disturbances (not shown).

Nonlinear Simulation Tests

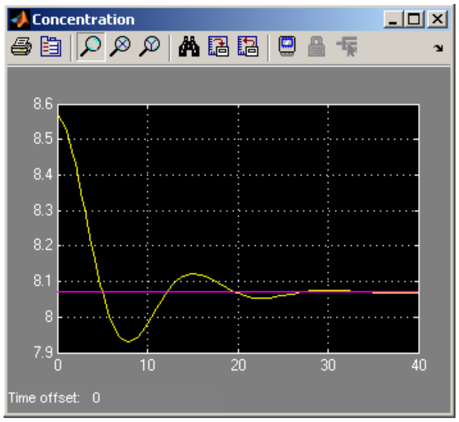
The controller seems to be performing well in linear tests, so try it with the nonlinear plant. As will be demonstrated below, you can run simulations that automatically use your latest design from the design tool. If the performance is unsatisfactory, you can easily change your design and retest.

In the `CSTR_MPC` model window, change the concentration setpoint to `9.07`. This simulates a 0.5-unit step increase in the setpoint at time t = 0. Run the Simulink simulation. The MPC Controller block automatically obtains the most recent **MPC1** definition from the design tool. If necessary, open the model's concentration scope block. The result should be as shown below.



The nonlinear response is more sluggish than the linear prediction. The controller reduces the coolant temperature to about 284.6 K (verify this by opening the model's coolant temperature scope), whereas the linear simulation predicts a reduction to 289.4 K. In other words, when plant moves in this direction, the linearized model's gain is too large. Still, the concentration goes to the setpoint rapidly.

Next, set the concentration setpoint to `8.07`, i.e., a step-change of equal magnitude in the opposite direction. Run the simulation to obtain the concentration scope response shown below.



In this direction, the response is underdamped. You can verify that the controller changes the coolant temperature to 304.1 K, an increase of 6.0 K (recall the decrease of 13.5 K when the change was in the opposite direction). In other words, the controller's linear model underpredicts the effect of a coolant temperature increase. If the setpoint were reduced significantly, the closed-loop system would become unstable (try `7.0`, for example).

Thus, the controller's effective operating range is limited. If you wanted to operate at a low concentration, you'd need to determine a linearized model at that condition and use it to design another controller.

Note This is typical of strongly nonlinear plants. If you needed to operate a conventional controller over such a wide range, you might consider gain scheduling. Equivalently, you could define predictive controllers for several operating points and switch from one to another depending on the measured concentration (see "Simulations Involving Nonlinear Plants" on page 4-9 for an example of this).

For the tested range, however, the oscillations die out quickly. You can verify that the controller responds equally well to small, sustained disturbances (i.e., ± 0.5 in feed concentration or ± 3 in feed temperature.

Modifying the Controller Using the Design Tool

Keeping the design tool open makes it easy to test controller modifications. Each time you run a Simulink simulation, the MPC Controller block automatically obtains the latest settings for its **MPC controller** parameter (**MPC1** in the above tests).

It's also easy to compare several controllers. Suppose you wanted to try a different control interval (the controller designed above uses a 1-second interval). Create a new controller by right-clicking the design tool's **MPC1** controller node, and selecting **Copy Controller** in the menu. This creates a duplicate controller called **MPC1_copy**. Rename it **MPC2**. Select **MPC2** in the tree, activate its **Model and Horizons** tab and set **Control interval** to 2.

Now return to the MPC Controller block mask (if necessary, double-click the block to open the mask) and change **MPC controller** from `MPC1` to `MPC2`. Close the mask or click the **Apply** button to notify the controller of this change. Run a simulation with a disturbance or setpoint change, and then open the **Coolant Temperature** scope. You should see that the controller is now making step-wise adjustments every 2 minutes. As would be expected, controller performance degrades with less frequent measurement feedback, but not much in this case.

To restore the original controller, just change `MPC2` to `MPC1` in the block mask and click **Apply**.

Exiting the Design Tool

When you close the design tool, you'll be prompted to save any projects you've created or edited. For details, see "Saving a Project" on page 3-62.

Once the design tool has closed, the MPC Controller block mask expects its controller object to be in your workspace. Thus, you'll also be prompted to export such controller objects. You need to do this if you plan to run your Simulink model after you've closed the design tool.

Saving Your Work

You'll usually want to save your design so you can reuse or revise it. You can save individual controllers or an entire project.

When you close the design tool, you'll be prompted to save new or modified designs. You can also save manually to preserve an intermediate state or guard against an unexpected shutdown.

This section covers the following topics:

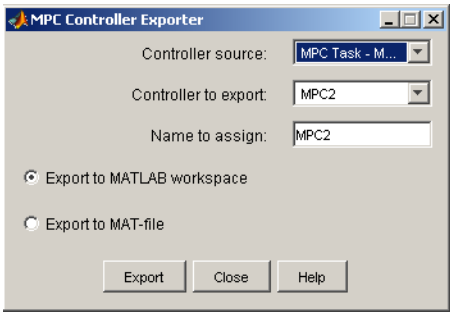
- "Exporting a Controller" on page 3-61
- "Saving a Project" on page 3-62

Exporting a Controller

To save a controller, *export* it to your MATLAB® workspace or to a MAT-file. The former allows you to use the exported controller in command-line functions or a Simulink® block.

Note Your workspace disappears when you exit MATLAB. A MAT-file is permanent, and you can reload it in a subsequent MATLAB session.

The following example assumes that the design tool is open and in the state described in the previous section. Suppose you want to export the controller to your workspace. Expand the tree if necessary, right-click **MPC2**, and select **Export Controller** from the resulting menu. The following dialog box appears.



MPC Controller Exporter

Controller source: MPC Task - M...

Controller to export: MPC2

Name to assign: MPC2

(●) Export to MATLAB workspace

( ) Export to MAT-file

The default behavior is to export the selected controller to the workspace. Click **Export** to confirm. You can verify the export by typing

```
whos
```

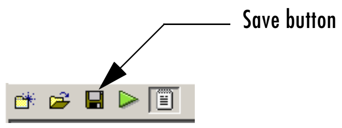
at the MATLAB prompt. The resulting list should include an `mpc` object named `MPC2`. Type

```
MPC2
```

to display the object's properties.

Saving a Project

To save your entire project, click the toolbar's Save button.



The following dialog box appears.

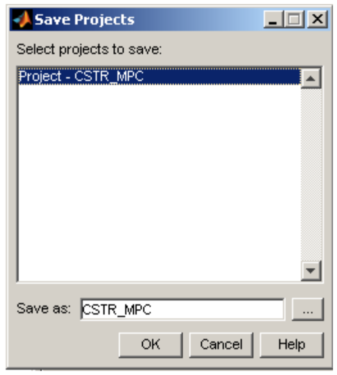


Figure 3-38: Dialog Box for Saving a Controller Design Project

The default behavior saves the current project (named `Project - CSTR_MPC` in this case) in a MAT-file (called `CSTR_MPC` here). If the design tool had contained other projects, they would appear in the list, and you could select the ones you wanted to save.

The MAT-file will be saved in the default directory. To verify the location, click the Browse (**...**) button and change the directory if necessary. When ready to save, click **OK**.

Loading Your Saved Work

The following assumes that you've saved a project as described in the previous section. To reload this project, close the design tool if it's open. Also clear any `mpc` objects from your workspace. (Type `whos` at the MATLAB® prompt for a list of objects.) For example, if `MPC1` and `MPC2` are in your workspace, type

```
clear MPC1 MPC2
```

to clear (remove) them.

If you've closed the `CSTR_MPC` model, open it. (Figure 3-36 shows the model diagram). Double-click the MPC Controller block to open its mask, and verify that the **MPC Controller** parameter is set to `MPC2`.

Note If you were to attempt to run the `CSTR_MPC` model at this stage, an error dialog box would indicate that the MPC Controller block was unable to initialize. The `MPC2` object specified in the block mask must be loaded into your workspace or be part of an active design tool task.

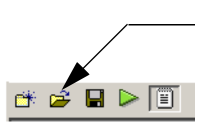
You could define the required `MPC2` object in one of the following ways:

- Import `MPC2` from a MAT-file (assuming you had saved it as explained in "Exporting a Controller" on page 3-61).
- Load the model's project file, which contains a copy of `MPC2`.

To use the second approach, open the design tool by typing

```
mpctool
```

in the MATLAB Command Window. This creates a blank Model Predictive Control Toolbox™ project called **MPCdesign**. Click the Load button on the toolbar.



This opens a dialog box similar to that shown in Figure 3-38. Use it to select the project file you've saved, and then click **OK** to load the project. It should appear in the tree. Verify that it contains a controller named `MPC2`.

Run the `CSTR_MPC` model in Simulink®. The block mask automatically retrieves **MPC2** from the design tool, and the simulation runs. In other words, loading the project automatically restores the link between the design tool and the MPC Controller block.

4

Using Functions

Control Based On Multiple Plant Models (p. 4-14)

Controller Definition

Chapter 3, “Designing the Controller Using the Design Tool GUI” showed how to use the Model Predictive Control Toolbox™ design tool to create a controller and test it. You might prefer to use functions instead. They allow access to options not available in the design tool, as well as automation of repetitive tasks using scripts and customized plotting.

This section covers the following topics:

- “Creating a Controller Object” on page 4-2
- “Viewing and Altering Controller Properties” on page 4-3

Creating a Controller Object

The following uses the `CSTR` model described in Chapter 2, “Building Models” as an example. To follow along, verify that the model’s LTI object is in your MATLAB® workspace (if necessary, create it as explained in “Chemical Reactor Example” on page 2-5, and set its label and signal type properties as explained in “LTI Properties for the CSTR Example” on page 2-7).

Use the `mpc` function to create a controller. For example, type

```
Ts = 1;
MPCobj = mpc(CSTR, Ts);
```

to create one based on the `CSTR` model with a control interval of 1 time unit and all other parameters at their default values.

Note `MPCobj` is an MPC object. It contains a complete controller definition for use with Model Predictive Control Toolbox software.

To display the controller’s properties in the Command Window, type

```
display(MPCobj)
```

or type the object’s name without a trailing semicolon.

Viewing and Altering Controller Properties

Once you've defined an MPC object, it's easy to alter its properties. For a description of the editable properties, type:

```
mpcprops
```

To display the list of properties and their current values, type

```
get(MPCobj)
```

For the CSTR example this displays:

```
ManipulatedVariables (MV): [1x1 struct]
        OutputVariables (OV): [1x2 struct]
   DisturbanceVariables (DV): [1x1 struct]
                 Weights (W): [1x1 struct]
                       Model: [1x1 struct]
                          Ts: 1
                   Optimizer: [1x1 struct]
       PredictionHorizon (P): 10
              ControlHorizon: 2
                     History: [2e+003 7 21 20 18 20.1]
                       Notes: {}
                    UserData: []
```

(Your `History` entry will differ.) To alter one of these properties, you can use the syntax

```
ObjName.PropName = value;
```

where `ObjName` is the object name, and `PropName` is the property you want to set. For example, to change the prediction horizon from 10 (the default) to 15, type:

```
MPCobj.P = 15;
```

Note You can abbreviate property names provided that the abbreviation is unambiguous.

As shown above, many of the properties are MATLAB *structures* containing additional properties. For example, type

```
MPCobj.MV
```

which displays:

```
          Min: -Inf
          Max: Inf
       MinECR: 0
       MaxECR: 0
      RateMin: -Inf
      RateMax: Inf
   RateMinECR: 0
   RateMaxECR: 0
       Target: 'nominal'
         Name: 'T_c'
        Units: ''
```

This shows that the default controller has no constraints on the manipulated variable. To include constraints as shown in Figure 3-17, you could type

```
MPCobj.MV.Min = -10;
MPCobj.MV.Max = 10;
MPCobj.MV.RateMin = -4;
MPCobj.MV.RateMax = 4;
MPCobj.MV.Units = 'Deg C';
```

or use the `set` command:

```
set(MPCobj, 'MV', struct('Min', -10, 'Max', 10, ...
   'RateMin', -4, 'RateMax', 4, 'Units', 'Deg C'));
```

Note The `Units` property is for display purposes only and is optional.

There are two outputs in this case, so `MPCobj.OV` is a 1-by-2 structure. To set measurement units to the values shown in Figure 3-11, you could type

```
MPCobj.OV(1).Units = 'Deg C';
MPCobj.OV(2).Units = 'kmol/m^3';
```

Finally, check the default weights by typing

```
MPCobj.W
```

which displays:

```
          ManipulatedVariables: 0
      ManipulatedVariablesRate: 0.1000
               OutputVariables: [1 1]
                           ECR: 100000
```

Change to the values shown in Figure 3-11 by typing:

```
MPCobj.W.ManipulatedVariablesRate = 0.3;
MPCobj.W.OutputVariables = [1 0];
```

You can also specify time-varying weights and constraints. The time-varying weights and constraints are defined for the prediction horizon, which shifts at each time step. This implies that as long as the property is not changed, the set of time-varying parameters is the same at each time step. Type `mpcprops` or see the User's Guide for details. To learn how to specify time-varying constraints and weights in the GUI, see "Constraints Tab" and "Weight Tuning Tab".

The time-varying weights modify the tuning of the unconstrained controller response. To specify a different weight for each step in the prediction horizon, modify the `Weight` property. For example,

```
MPCobj.W.OutputVariables = [0.1 0; 0.2 0; 0.5 0; 1 0];
```

deemphasizes setpoint tracking errors early in the prediction horizon. The default weight of 1 is used for the fourth step and beyond.

Constraints can also be time varying. The time-varying constraints have a nonlinear effect when they are active. For example,

```
MPCobj.MV.RateMin=[-4;-3.5;-3;-2.5]
MPCobj.MV.RateMax=[4;3.5;3;2.5]
```

forces `MV` to change more and more slowly along the prediction horizon. The constraint of -2.5 and 2.5 is used for the fourth step and beyond.

You could also alter the controller's disturbance rejection characteristics using functions that parallel the design tool's disturbance modeling options (described in "Disturbance Modeling and Estimation" on page 3-29). See the reference pages for the `setestim`, `setindist`, and `setoutdist` functions.

Linear Simulations

Model Predictive Control Toolbox™ functions allow you to perform linear closed-loop and open-loop simulations. This section covers the following topics:

- "Using the sim Function" on page 4-7
- "Saving Calculated Results" on page 4-7
- "Simulation Options" on page 4-8

Using the sim Function

To run a linear simulation, use the `sim` function. For example, given the `MPCobj` controller defined in the previous section, type:

```
T = 26;
r = [2 0];
sim(MPCobj, T, r);
```

This simulates the closed-loop response for a duration of 26 control intervals with a setpoint of 2 for the first output (the reactor temperature) and 0 for the second output (the residual concentration). Recall that the second output's tuning weight is zero (see the discussion in "Output Weights" on page 3-20), so its setpoint is ignored.

By default, the same linear model is used for controller predictions and the plant, i.e., there is no plant/model mismatch. You can alter this as shown in "Simulation Options" on page 4-8.

When you use the above syntax (no output variables), `sim` automatically plots the plant inputs and outputs (not shown, but see Figure 3-12 and Figure 3-13 for results of a similar scenario).

Saving Calculated Results

If you'd like to save simulation results in your workspace, use the following `sim` function format:

```
[y, t, u] = sim(MPCobj, T, r);
```

This suppresses automatic plotting, instead creating variables `y`, `t`, and `u`, which hold the computed outputs, time, and inputs, respectively. A typical use is to create customized plots. For example, to plot both outputs on the same axis versus time, you could type:

```
plot(t, y)
```

Simulation Options

You can modify simulation options using the `mpcsimopt` function. For example, the code

```
MPCopts = mpcsimopt;
MPCopts.Constraints = 'off';
sim(MPCobj, T, r, MPCopts)
```

runs an unconstrained simulation. Comparing to the case described in “Using the sim Function” on page 4-7, the controller’s first move is now exceeds 4 units (the specified rate constraint).

Other options include the addition of a specified noise sequence to the manipulated variables or measured outputs, open-loop simulations, a look-ahead option for better setpoint tracking or measured disturbance rejection, and plant/model mismatch.

For example, the following code defines a new plant model having gains 50% larger than those in the `CSTR` model used in the controller, then repeats the above simulation:

```
Plant = 1.5*CSTR;
MPCopts.Model = Plant;
sim(MPCobj, T, r, MPCopts)
```

In this case, the plant/model mismatch degrades controller performance, but only slightly. Degradation can be severe and must be tested on a case-by-case basis.

Simulations Involving Nonlinear Plants

You can also use `sim` to simulate a closed-loop system consisting of a linear plant model and an MPC controller.

If your plant is a nonlinear Simulink® model, you must linearize the plant (see “Linearization Using Simulink® Control Design™” on page 2-19) and design a controller for the linear model (see “Nonlinear Plants” on page 3-50). To simulate the system, specify the controller in the MPC block parameter **MPC Controller** field and run the closed-loop Simulink model.

Alternatively, your nonlinear model might be a MEX-file, or you might want to include features unavailable in the MPC block, such as a custom state estimator. The `mpcmove` function is the Model Predictive Control Toolbox™ computational engine, and you can use it in such cases. The disadvantage is that you must duplicate the infrastructure that the `sim` function and the MPC block provide automatically.

The rest of this section covers the following topics:

- “Nonlinear CSTR Application” on page 4-9
- “Example Code for Successive Linearization” on page 4-10
- “CSTR Results and Discussion” on page 4-11

Nonlinear CSTR Application

The `CSTR` model described in “Using Simulink® to Develop LTI Models” on page 2-19 is a strongly nonlinear system. As shown in “Nonlinear Plants” on page 3-50, a controller can regulate this plant, but degrades (and might even become unstable) if the operating point changes significantly.

The objective of this example is to redefine the predictive controller at the beginning of each control interval so that its predictive model, though linear, represents the latest plant conditions as accurately as possible. This will be done by linearizing the nonlinear model repeatedly, allowing the controller to adapt as plant conditions change. See references [1] and [2] for more details on this approach.

Example Code for Successive Linearization

In the following code, the simulation begins at the CSTR model's nominal operating point (concentration = 8.57) and moves to a low concentration (= 2) where the reaction rate is much higher. The required code is as follows:

```
[sys, xp] = CSTR_INOUT([],[],[],'sizes');
up = [10 298.15 298.15];
u = up(3);
tsave = []; usave = []; ysave = []; rsave = [];
Ts = 1;
t = 0;
while t < 40
    yp = xp;
    % Linearize the plant model at the current conditions
    [a,b,c,d]=linmod('CSTR_INOUT', xp, up );
    Plant = ss(a,b,c,d);
    Plant.InputGroup.ManipulatedVariables = 3;
    Plant.InputGroup.UnmeasuredDisturbances = [1 2];
    Model.Plant = Plant;
    % Set nominal conditions to the latest values
    Model.Nominal.U = [0 0 u];
    Model.Nominal.X = xp;
    Model.Nominal.Y = yp;
    dt = 0.001;
    Options = simset('InitialState', xp);
    [T, XP, YP] = sim('CSTR_INOUT', [t t+dt], Options, ...
        [t up; t+dt up]);
    Model.Nominal.DX = (1/dt)*(XP(end,:)' - xp(:));
    % Define MPC Toolbox controller for the latest model
    MPCobj = mpc(Model, Ts);
    MPCobj.W.Output = [0 1];
    % Ramp the setpoint
    r = max([8.57 - 0.25*t, 2]);
    % Compute the control action
    if t <= 0
        xd = [0; 0];
        x = mpcstate(MPCobj, xp, xd, [], u);
    end
    u = mpcmove(MPCobj, x, yp, [0 r], []);
    % Simulate the plant for one control interval
```

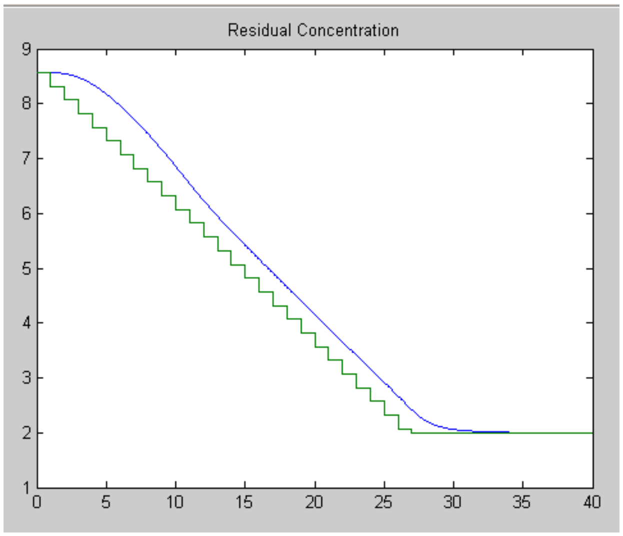
```
        up(3) = u;
        Options = simset('InitialState', xp);
        [T, XP, YP] = sim('CSTR_INOUT', [t t+Ts], Options, ...
            [t up; t+Ts up]);
        % Save results for plotting
        tsave = [tsave; T];
        ysave = [ysave; YP];
        usave = [usave; up(ones(length(T),1),:)];
        rsave = [rsave; r(ones(length(T),1),:)];
        xp = XP(end,:)';
        t = t + Ts;
    end
    figure(1)
    plot(tsave,[ysave(:,2) rsave])
    title('Residual Concentration')
    figure(2)
    plot(tsave,usave(:,3));
    title('Coolant Temperature')
```

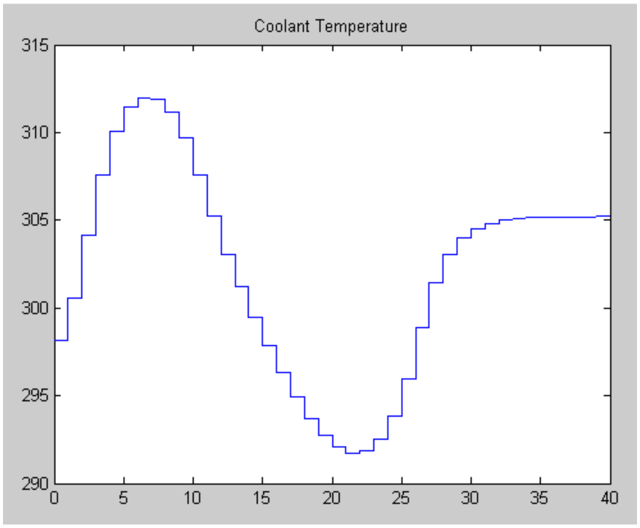
CSTR Results and Discussion

The plotted results appear below. Note the following points:

- The setpoint is being ramped from the initial concentration to the desired final value (see the step-wise changes in the reactor concentration plot below). The reactor concentration tracks this ramp smoothly with some delay (see the smooth curve), and settles at the final state with negligible overshoot. The controller works equally well (and achieves the final concentration more rapidly) for a step-wise setpoint change, but it makes unrealistically rapid changes in coolant temperature (not shown).

- The final steady state requires a coolant temperature of 305.20 K (see the coolant temperature plot below). An interesting feature of this nonlinear plant is that if one starts at the initial steady state (coolant temperature = 298.15 K), stepping the coolant temperature to 305.20 and holding will not achieve the desired final concentration of 2. In fact, under this simple strategy the reactor concentration stabilizes at a final value of 7.88, far from the desired value! A successful controller must increase the reactor temperature until the reaction “takes off,” after which it must reduce the coolant temperature to handle the increased heat load. The relinearization approach provides such a controller (see following plots).





- Function `linmod` relinearizes the plant as its state evolves. This function was discussed previously in "Linearization Using Simulink® Functions" on page 2-24.
- The code also resets the linear model's nominal conditions to the latest values. Note, however, that the first two input signals, which are unmeasured disturbances in the controller design, always have nominal zero values. As they are unmeasured, the controller cannot be informed of the true values. A non-zero values would cause an error.
- Function `mpc` defines a new controller based on the relinearized plant model. The output weight tuning ignores the temperature measurement, focusing only on the concentration.
- At $t = 0$, the `mpcstate` function initializes the controller's extended state vector, `x`, which is an *mpcstate object*. Thereafter, the `mpcmove` function updates it automatically using the controller's default state estimator. It would also be possible to use an Extended Kalman Filter (EKF) as described in [1] and [2], in which case the EKF would reset the `mpcstate` input variables at each step.
- The `mpcmove` function uses the latest controller definition and state, the measured plant outputs, and the setpoints to calculate the new coolant temperature at each step.
- The Simulink `sim` function simulates the nonlinear plant from the beginning to the end of the control interval. Note that the final condition from the previous step is being used as the initial plant state, and that the plant inputs are being held constant during each interval.

Remember that a conventional feedback controller or a fixed Model Predictive Control Toolbox controller tuned to operate at the initial condition would become unstable as the plant moves to the final condition. Periodic model updating overcomes this problem automatically and provides excellent control under all conditions.

Control Based On Multiple Plant Models

The “Nonlinear CSTR Application” on page 4-9 shows how updates to the prediction model can improve MPC performance. In that case the model is nonlinear and you can obtain frequent updates by linearization.

A more common situation is that you have several linear plant models, each of which applies at a particular operating condition and you can design a controller based on each linear model. If the models cover the entire operating region and you can define a criterion by which you switch from one to another as operating conditions change, the controller set should be able to provide better performance than any individual controller.

The Model Predictive Control Toolbox® includes a Simulink® block that performs this function. It is the *Multiple MPC Controllers* block. The rest of this section is an illustrative example organized as follows:

- “A Two-Model Plant” on page 4-14
- “Designing the Two Controllers” on page 4-16
- “Simulating Controller Performance” on page 4-17

A Two-Model Plant

Note The `mpcswitching` demo provides an animated version of the plant described below.

Figure 4-1 is a stop-action snapshot of the subject plant. It consists of two masses, M_1 and M_2. A spring connects M_1 to a rigid wall and pulls it to the right. An applied force, shown as a red arrow in Figure 4-1, opposes this spring, pulling M_1 to the left.

When the two masses are detached, as in Figure 4-1, mass M_2 is uncontrollable and responds only to the spring pulling it to the left.

If the two masses collide, however, they stick together (the collision is completely inelastic) until a change in the applied force separates them.

The control objective is to move M_1 in response to a command signal. The blue triangle in Figure 4-1 represents the desired location. At the instant shown, the desired location is -5.

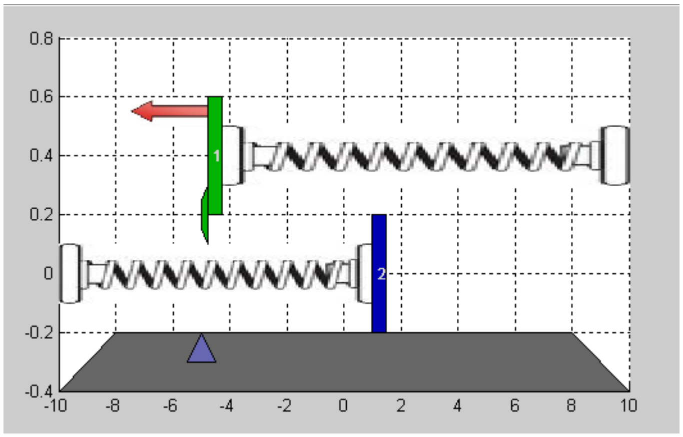


Figure 4-1: Animation of the Multi-Model Example

In order to achieve its objective, the controller can adjust the applied force magnitude (the length of the red arrow). It receives continuous feedback on the M_1 location. There is also a contact sensor to signal collisions. The M_2 location is unmeasured.

If M_1 were isolated, this would be a routine control problem. The challenge is that the relationship between the applied force and the M_1 movement changes dramatically when M_2 attaches to M_1.

The following code defines the model. First define the system parameters as follows:

```
%% Model Parameters
M1=1;       % mass
M2=5;       % mass
k1=1;       % spring constant
k2=0.1;     % spring constant
b1=0.3;     % friction coefficient
b2=0.8;     % friction coefficient
yeq1=10;    % wall mount position
yeq2=-10;   % wall mount position
```

Next define a model of M_1 when the masses are separated. Its states are the M_1 position and velocity. Its inputs are the applied force, which will be the controller's manipulated variable, and a spring constant calibration signal, which is a measured disturbance input.

```
A1=[0 1;-k1/M1 -b1/M1];
B1=[0 0;-1/M1 k1*yeq1/M1];
C1=[1 0];
D1=[0 0];
sys1=ss(A1,B1,C1,D1);
sys1=setmpcsignals(sys1, 'MV', 1, 'MD', 2);
```

The `setmpcsignals` command specifies the input type for the two inputs.

We need another model (with the same input/output structure) to predict movement when the two masses are joined, as follows:

```
A2=[0 1;-(k1+k2)/(M1+M2) -(b1+b2)/(M1+M2)];
B2=[0 0;-1/(M1+M2) (k1*yeq1+k2*yeq2)/(M1+M2)];
C2=[1 0];
D2=[0 0];
sys2=ss(A2,B2,C2,D2);
sys2=setmpcsignals(sys2, 'MV', 1, 'MD', 2);
```

Designing the Two Controllers

Next we define controllers for each case. Both use a 0.2 second sampling period, a prediction horizon of $P = 20$, a control horizon of $M = 1$, and the default values for all other controller design parameters. The only difference in the controllers is the prediction model.

```
Ts=0.2;     % sampling time
p=20;       % prediction horizon
m=1;        % control horizon
MPC1=mpc(sys1,Ts,p,m); % Controller for M1 detached from M2
MPC2=mpc(sys2,Ts,p,m); % Controller for M1 connected to M2
```

The applied force also has the same constraints in each case. Its lower bound is zero (it can't reverse direction), and its maximum rate of change is 1000 per second (increasing or decreasing).

```
MPC1.MV=struct('Min',0,'RateMin',-1e3,'RateMax',1e3);
MPC2.MV=struct('Min',0,'RateMin',-1e3,'RateMax',1e3);
```

Simulating Controller Performance

Figure 4-2 shows the Simulink block diagram for this example. The upper portion simulates the movement of the two masses, plots the signals as a function of time, and animates the demo.

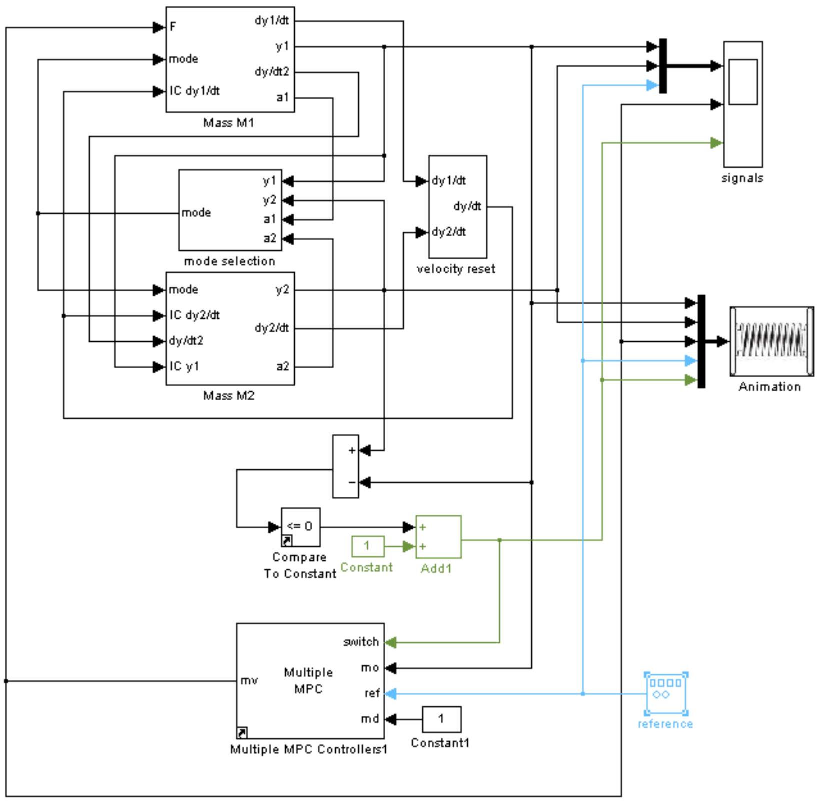


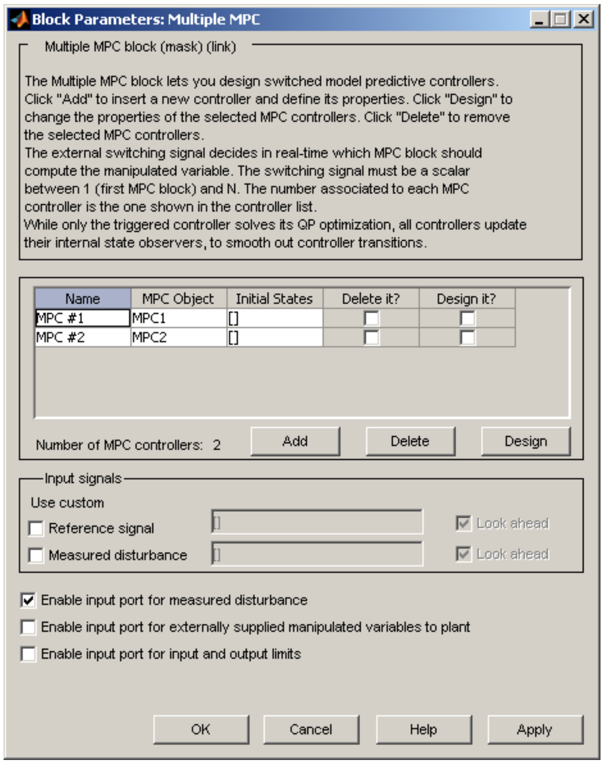
Figure 4-2: Block Diagram of the Two-Model Example

The lower part contains three key elements:

- A pulse generator that supplies the desired M_1 position (the controller reference signal). Its output is a square wave varying between -5 and 5 with a frequency of 0.015 per second.

- A simulation of a contact sensor. When the two masses have the same position, the `Compare to Constant` block evaluates to `true`, and the `Add1` block converts this to a 2. Otherwise, the `Add1` output is 1.
- The Multiple MPC Controller block. It has four inputs. The measured output (`mo`), reference (`ref`), and measured disturbance (`md`) inputs are as for a standard MPC Controller block. The distinctive feature is the `switch` input.

The figure below shows the Multiple MPC Controller block mask for this example (obtained by double clicking on the controller block).



Block Parameters: Multiple MPC

Multiple MPC block (mask) (link)

The Multiple MPC block lets you design switched model predictive controllers. Click "Add" to insert a new controller and define its properties. Click "Design" to change the properties of the selected MPC controllers. Click "Delete" to remove the selected MPC controllers.
The external switching signal decides in real-time which MPC block should compute the manipulated variable. The switching signal must be a scalar between 1 (first MPC block) and N. The number associated to each MPC controller is the one shown in the controller list.
While only the triggered controller solves its QP optimization, all controllers update their internal state observers, to smooth out controller transitions.

Name	MPC Object	Initial States	Delete it?	Design it?
MPC #1	MPC1	[]	☐	☐
MPC #2	MPC2	[]	☐	☐

Number of MPC controllers: 2

Input signals
Use custom
☐ Reference signal [] ☑ Look ahead
☐ Measured disturbance [] ☑ Look ahead

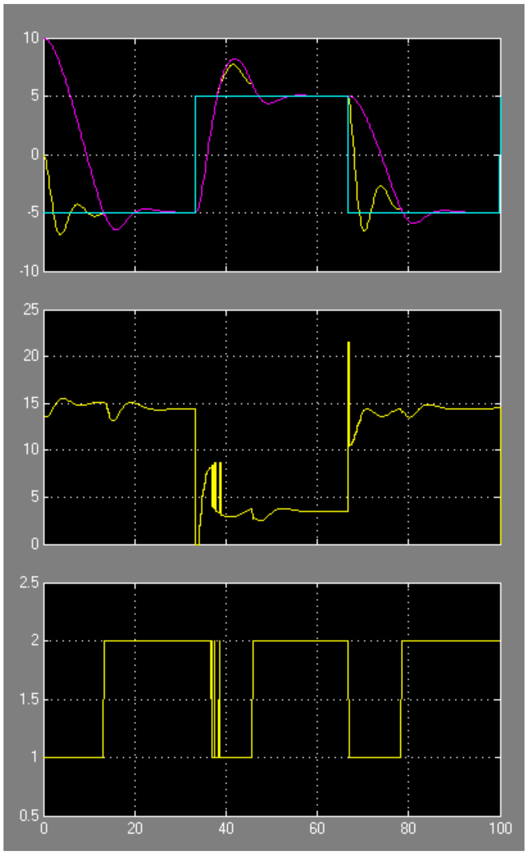
☑ Enable input port for measured disturbance
☐ Enable input port for externally supplied manipulated variables to plant
☐ Enable input port for input and output limits

When the `switch` input is 1 the block automatically activates the first controller listed (`MPC1`), which is appropriate when the masses are separated. When the `switch` input is 2 the block automatically enables the second controller (`MPC2`).

The following code simulates the controller performance

```
Tstop=100;  % Simulation time
y1initial=0; % Initial M1 and M2 Positions
y2initial=10;
open('mpc_switching');
sim('mpc_switching',Tstop);
```

The figure below shows the `signals` scope output.



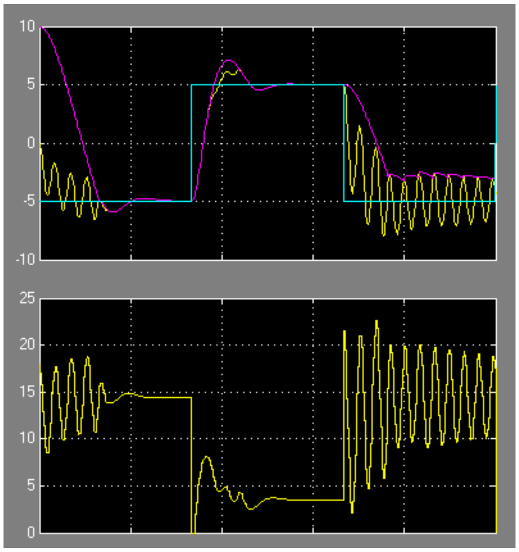
In the upper plot, the cyan curve is the desired position. It starts at -5. The M_1 position (yellow) starts at 0 and under the control of `MPC1`, M_1 moves rapidly toward the desired position. M_2 (magenta) starts at 10 and begins moving in the same direction. At about t = 13 seconds, M_2 collides with M_1. The switching signal (lower plot) changes at this instant from 1 to 2, so controller `MPC2` has taken over.

The collision moves M_1 away from its desired position and M_2 remains joined to M_1. Controller `MPC2` adjusts the applied force (middle plot) so M_1 quickly returns to the desired position.

When the desired position changes step-wise to 5, the two masses separate briefly (with appropriate switching to `MPC1`) but for the most part move together and settle rapidly at the desired position. The transition back to -5 is equally well behaved.

Now suppose we force `MPC2` to operate under all conditions. The figure below shows the result. When the masses are separated, as at the start, `MPC2` applies excessive force and then over-compensates, resulting in oscillatory behavior. Once the masses join, the movement smooths out, as would be expected.

The oscillations are especially severe in the last transition. The masses collide frequently and M_1 never reaches the desired position.



If we put `MPC1` in charge exclusively, we instead see sluggish movements that fail to settle at the desired position before the next transition occurs. (not shown but you can run the `mpcswitching` demo).

In this case, at least, two controllers are better than one!

Analysis Tools

The are many ways to analyze a controller design. This section highlights two functions that support analysis of Model Predictive Control Toolbox™ controllers:

- "Steady-State Gain Computation" on page 4-22
- "Controller Extraction" on page 4-22

Steady-State Gain Computation

The `cloffset` function computes the closed-loop, steady-state gain for each output when subjected to a sustained, 1-unit disturbance added to each output. It assumes that no constraints will be encountered.

For example, consider the controller operating at the final steady-state of the nonlinear `CSTR` of the previous section. To compute its gain, type

```
cloffset(MPCobj)
```

which gives the result:

```
ans =

     1.0000   14.5910
    -0.0000   -0.0000
```

The interpretation is that the controller doesn't react to a sustained disturbance of 1 unit in the first output (the reactor temperature). Recall that we assigned zero weight to this output in the controller design, so the controller ignores deviations from its setpoint. The same disturbance has no effect on the second output (the 2,1 element is zero).

If there is a 1-unit disturbance in the second output, the controller reacts, and the first output increases 14.59 units. This is again due to the zero weight on this output. The second output stays at its setpoint (the 2,2 element is zero).

Controller Extraction

Use the `ss` function to obtain an LTI representation of an unconstrained Model Predictive Control Toolbox controller. You can use this to analyze the controller's closed-loop frequency response, etc.

For example, consider the controller designed in "Creating a Controller Object" on page 4-2. To extract the controller, you could type:

```
MPCss = ss(MPCobj);
```

You could then construct an LTI model of the closed-loop system using the `feedback` function (see the Control System Toolbox™ documentation for details) by typing:

```
CSTRd = c2d(CSTR, MPCss.Ts);
Feedin = 1;
Feedout = 1;
Sign = 1;
CLsys = feedback(CSTRd, MPCss, Feedin, Feedout, Sign);
```

Note The `CSTR` model must be converted to discrete form with the same control interval as the controller.

Recall that the CSTR plant has two inputs and two outputs. The first input is the manipulated variable and the other is an unmeasured disturbance. The first output is measured for feedback and the other is not. The `Feedin` and `Feedout` parameters specify the input and output to be used for control. The `Sign` parameter signifies that the MPC object uses positive feedback, i.e., the measured outputs enter the controller with no sign change. Omission of this would cause the `feedback` command to use negative feedback by default and would almost certainly lead to an unstable closed-loop system.

You could then type

```
eig(CLsys)
```

to verify that all closed-loop poles are within the unit circle, or

```
bode(CLsys)
```

to compute a closed-loop bode plot.

Bibliography

[1] Lee, J. H. and N. L. Ricker, “Extended Kalman Filter Based Nonlinear Model Predictive Control,” *Ind. Eng. Chem. Res.*, Vol. 33, No. 6,pp. 1530-1541 (1994).

[2] Ricker, N. L., and J. H. Lee “Nonlinear Model Predictive Control of the Tennessee Eastman Challenge Process,” *Computers & Chemical Engineering*, Vol. 19, No. 9, pp. 961-981 (1995).

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